MTH C

Solved Exams

وفرها لكم محمود صفوت

2012/2013

هذا العمل الخيري صدقة لروح اخونا ايمن الجرجاوي نسالكم الفاتحة و الدعاء له بالرحمة

مؤلف هذه الملذمة ليس من الطلاب الاحرار انما هو دكتور يدرس للطلاب بالخارج لا يسمح لنا باظهار اسمة لعدم الدعاية الغير المصر بها

Higher Technological Institute
10th Of Ramadan City
Department Of Basic Sciences

Subject : MATH (C) (MTH101) — Term : Sep /Jan 2010-2011

Examiner : Examiners Committee — Time : 90 min

Final Written Examination

Answer the following questions :

1- (a) Use the power series representation for e^x to find approximate value for $\int_0^{0.1} e^{-x^2}\,dx$. (3 Points)

(b) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ if :

$$w = e^{xyz} \quad , \quad x = r + s + t \quad , \quad y = rst \quad , \quad z = r^2 + s^2 + t^2 .$$ (4 Points)

(c) For the cardioid $r = 2 + 2\cos\theta$ with $0 \le \theta \le 2\pi$. Find the points at which the tangent line is horizontal . (3 Points)

2- (a) Sketch the graph of each of the following polar equations :

(i) $r = \dfrac{16}{4 - 4\sin\theta}$, (ii) $r = 6\sin 3\theta$. (4 Points)

(b) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ if :

$$w = e^{xyz} \quad , x = r + s + t \quad , y = rst \quad , \quad z = r^2 + s^2 + t^2 .$$ (4 Points)

(c) Evaluate $\int_{-1}^{1}\int_{y}^{3} y^2\, x\, dx\, dy$. (2 Points)

3- (a)Use partial differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the function :

$$xe^{yz} + ye^{xz} = z$$ (3 Points)

(b) Determine whether the following series converges or diverges :

(i) $\sum \dfrac{3n^2 + 5n}{2^n(n^2+1)}$, (ii) $\sum \dfrac{3^n}{n!}$. (4 Points)

c) Find the area of the region bounded by the cardioids $r = 2 + 2\cos\theta$. (3 Points)

Math ©

final Sep-Jan 2010/2011

① $e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots$

$\therefore e^{-x^2} = 1 + \frac{(-x^2)}{1!} + \frac{(-x^2)^2}{2!} + \frac{(-x^2)^3}{3!} \cdots$

$= 1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} \cdots$

$\therefore \int_0^{0.1} e^{-x^2} dx = \int_0^{0.1} 1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} \cdots dx$

$= x - \frac{x^3}{3(1!)} + \frac{x^5}{5(2!)} - \frac{x^7}{7(3!)} \cdots \Big|_0^{0.1}$

$\boxed{2}$

$w = e^{xyz}$ $\quad x = r+s+t$

$y = rst$ $\quad z = r^2+s^2+t^2$

الحل

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$$\Rightarrow e^{xyz}(yz)\cdot(1) + e^{xyz}(xz)\cdot(st)$$

$$+ e^{xyz}(xy)\cdot(2r)$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial s}$$

$$= e^{xyz}(yz)(1) + e^{xyz}(xz)\cdot(rt)$$

$$+ e^{xyz}(xy)(2s)$$

$r = 2 + 2\cos\theta$

اشتق

$\frac{dr}{d\theta} = -2\sin\theta$

$\therefore m = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

$m = \dfrac{(-2\sin\theta)\sin\theta + (2+2\cos\theta)\cos\theta}{(-2\sin\theta)\cos\theta - (2+2\cos\theta)\sin\theta}$

to be horizontal $y' = 0$ $\boxed{m = 0}$

$\therefore -2\sin^2\theta + 2\cos\theta + 2\cos^2\theta = 0$

$\boxed{\sin^2\theta = 1 - \cos^2\theta}$

$\Rightarrow -2(1-\cos^2\theta) + 2\cos\theta + 2\cos^2\theta = 0$

$$-2 + 2\cos^2\theta + 2\cos\theta + 2\cos^2\theta = 0$$

$$4\cos^2\theta + 2\cos\theta - 2 = 0$$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

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$2\cos\theta - 1 = 0$	$\cos\theta + 1 = 0$
$2\cos\theta = 1$	$\cos\theta = -1$
$\cos\theta = \frac{1}{2}$	$\theta = \pi$
$\theta = \frac{\pi}{3}$	$r = 2 + 2\cos\pi$

$\Rightarrow r = 2 + 2\cos\frac{\pi}{3}$

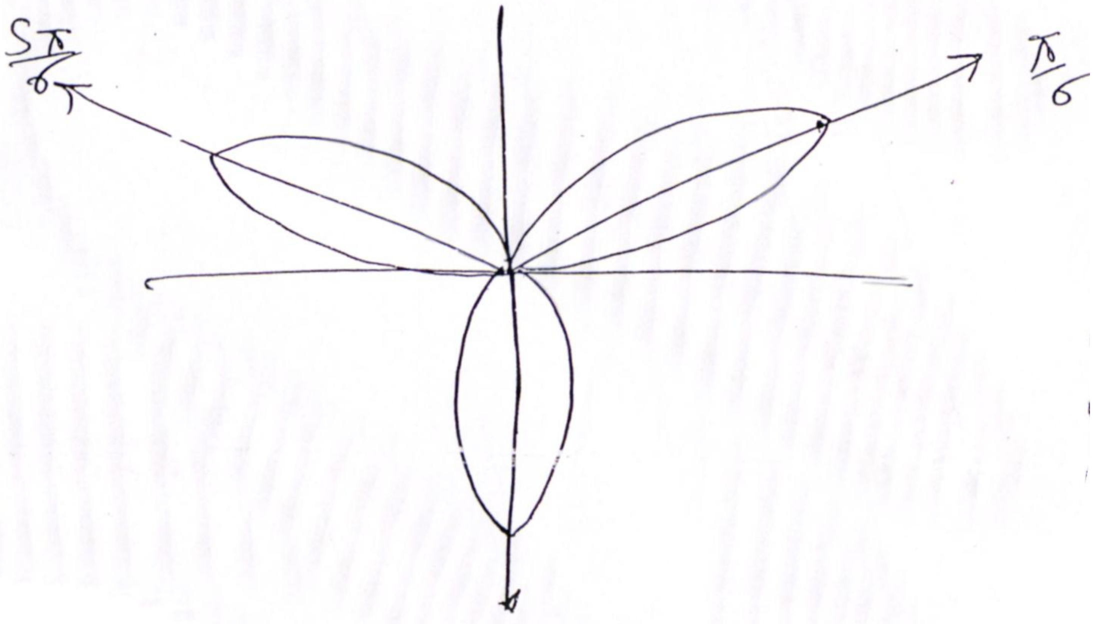
or $\theta = 2\pi - \frac{\pi}{3}$

$\Rightarrow r = 2 + 2\cos\left(2\pi - \frac{\pi}{3}\right)$

$r = 6 \sin 3\theta$

الحل

θ	0	-			2π
r					



السؤال مكرر المتحد أصول

(c) $\int_{-1}^{1}\int_{y}^{3} y\,x\,dx\,dy$

الحل

$$\int_{-1}^{1} y\,\frac{x^2}{2}\Big|_{y}^{3}\,dy$$

$$= \int_{-1}^{1} \frac{9}{2}y - \frac{y^3}{2}\,dy$$

$$= \frac{9}{2}\,\frac{y^2}{2} - \frac{y^4}{8}\Big|_{-1}^{1}$$

$$= 0 \quad \#$$

$$xe^{yz} + ye^{xz} = z$$

الكل

$$f = xe^{yz} + ye^{xz} - z = 0$$

$$f_x = e^{yz} + ye^{xz}z$$

$$f_y = xe^{yz}z + e^{xz}$$

$$f_z = xe^{yz}y + ye^{xz}x - 1$$

$$\frac{\partial z}{\partial x} = \frac{-f_x}{f_z}$$

$$\frac{\partial z}{\partial y} = \frac{-f_y}{f_z}$$

$$\sum \frac{3n^2+5n}{2^n(n^2+1)}$$

الحل

$$a_n = \frac{3n^2+5n}{2^n(n^2+1)} \qquad b_n = \frac{n^2}{2^n n^2} = \frac{1}{2^n} = \left(\frac{1}{2}\right)^n$$

$$\sum \left(\frac{1}{2}\right)^n \Rightarrow r = \frac{1}{2} < 1 \text{ conv}$$

$$\lim_{n\to\infty} \frac{3n^2+5n}{\cancel{2^n}(n^2+1)} * \frac{\cancel{2^n}}{1} = 3$$

$\therefore b_n \longrightarrow$ conv

$\therefore a_n \longrightarrow$ conv

$$\sum \frac{3^n}{n!}$$

الحل

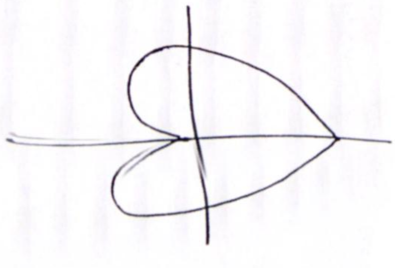
$$a_n = \frac{3^n}{n!} \qquad a_{n+1} = \frac{3^{n+1}}{(n+1)!}$$

$$\lim_{n\to\infty} \left| \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{3^{n+1}}{3^n} \cdot \frac{\cancel{n!}}{(n+1)\cancel{n!}} \right|$$

$$\lim_{n\to\infty} \left| \frac{3}{n+1} \right| = \frac{3}{\infty} = 0 < 1 \quad \text{Con}$$

$r = 2 + 2\cos\theta$



محاولة الرسم بالضبط

$f = \ln x \cos yz$

الحل

$f_x = \frac{1}{x} \cos yz$

$f_y = \ln x \, (-\sin yz) \cdot z$

$f_z = \ln x \, (-\sin yz) \, y$

HIGHER TECHNOLOGICAL INSTITUTE
TENTH OF RAMADAN CITY
DEPARTMENT OF BASIC SCIENCE
Final Exam

Subject: Math C (MTH 101) | Time: 90 minutes
Examiner: Board of Examiners | First Term: 2009-2010

الطلاب الأحرار

Answer the following questions:

Q1. (a) Study the convergence of the following series

(i) $\sum_{n=0}^{\infty} \frac{n^n}{n!}$ (ii) $\sum_{n=0}^{\infty} \frac{2n+3}{n^3+10}$

(b) Calculate the arc length of the curve defined by parametric equations

$x = 2(t - \sin t)$, $y = 2(1 - \cos t)$, $0 \le t \le 2\pi$

Q2. (a) Determine the interval and radius of convergence of

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{n5^n}$$

(b) Sketch the graph and calculate the area of

$r = 2(1 + \cos\theta)$, $0 \le \theta \le \pi$

$2+2\cos\theta$

Q3. (a) If $3z^3 = x^2z^2 - 2xy^3$. Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$.

(b) Evaluate the integral $\int_1^2 \int_x^{x^2} xy\,dy\,dx$, sketch the region of integration.

Q4. (a) Write only the expansion of the function $tan^{-1}x$ and then approximate $tan^{-1}0.2$.

(b) Use polar coordinate to evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \sin(x^2 + y^2)\,dy\,dx$,

With The Best Wishes,,,

الطلاب الأحرار

Revision ①

Limatals

① $\sum \frac{n^n}{n!}$

الحل

$a_n = \frac{n^n}{n!}$, $a_{n+1} = \frac{(n+1)^{n+1}}{(n+1)!}$

$\lim_{n\to\infty} \left| \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} \right|$

$\lim_{n\to\infty} \left| \frac{(n+1)^n \cancel{(n+1)}}{n^n} \cdot \frac{\cancel{n!}}{\cancel{(n+1)n!}} \right|$

$\lim_{n\to\infty} \left| \left(\frac{n+1}{n}\right)^n \right| = e > 1$

div

$$\sum \frac{2n+3}{n^3+10}$$

الحل

$a_n = \frac{2n+3}{n^3+10}$ $b_n = \frac{n}{n^3} = \frac{1}{n^2}$

$\therefore \sum \frac{1}{n^2}$ $p = 2 > 1$ Conv

$\lim_{n\to\infty} \frac{2n+3}{n^3+10} * \frac{n^2}{1} = 2$ (رقم)

$\therefore b_n \longrightarrow$ Conv

$\therefore a_n \longrightarrow$ Conv

(b) $X = 2(t - \sin t) \rightarrow \frac{dx}{dt} = 2(1-\cos t)$

$y = 2(1-\cos t) \rightarrow \frac{dy}{dt} = 2(\sin t)$

$0 < t < 2\pi$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$\int_0^{2\pi} \sqrt{(2(1-\cos t))^2 + (2\sin t)^2}\, dt$$

$$\int_0^{2\pi} 2\sqrt{1 - 2\cos t + \underbrace{\cos^2 t + \sin^2 t}_{1}}\, dt$$

$$2\int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt = 2\int_0^{2\pi} \sqrt{2(1-\cos t)}\, dt$$

$$2\int_0^{2\pi} \sqrt{2\left(2\sin^2 \frac{t}{2}\right)}\, dt$$

$$4\int_0^{2\pi} \sin \frac{t}{2}\, dt = 4\left(\frac{-\cos \frac{t}{2}}{\frac{1}{2}}\right)\Big|_0^{2\pi} = 16$$

٢١ $\sum \frac{(x-5)^n}{n\,5^n}$

الحل

$a_n = \frac{(x-5)^n}{n\,5^n}$ $\qquad a_{n+1} = \frac{(x-5)^{n+1}}{(n+1)\,5^{n+1}}$

$$\lim_{n\to\infty} \left| \frac{(x-5)^{n+1}}{(n+1)\,5^{n+1}} * \frac{n\,5^n}{(x-5)^n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{(x-5)^{n+1}}{(x-5)^n} \cdot \frac{n}{n+1} \cdot \frac{5^n}{5^{n+1}} \right| < 1$$

$\frac{n}{n+1} \to 1$

$$\Rightarrow \left| \frac{x-5}{5} \right| < 1 \qquad \times 5$$

$$|x-5| < 5$$

$$-5 < x-5 < 5 \qquad +5$$

$$0 < x < 10$$

to test end points

at $x = 0$ $\sum \frac{(-5)^n}{n5^n}$

$= \sum \frac{\cancel{5^n}(-1)^n}{n\,\cancel{5^n}} = \sum (-1)^n \frac{1}{n}$

(1) $\lim_{n \to \infty} \frac{1}{n} = 0$

(2) $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2}$ $f'(1) = -1 < 0$

Conv

at $x = 10$

$\sum \frac{\cancel{5^n}}{n\,\cancel{5^n}} = \sum \frac{1}{n}$ $P = 1$ div

Interval $0 \le x < 10$

Radius $R = 5$

0 10

$R = 5$

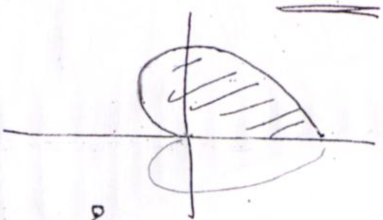
٤

(٤) $r = 2(1+\cos\theta)$

$0 \leq \theta \leq \pi$

نصف القلب

$$A = \frac{1}{2}\int_0^{\pi} r^2\,d\theta$$



$$= \frac{1}{2}\int_0^{\pi} \left(2(1+\cos\theta)\right)^2 d\theta$$

$$= 2\int_0^{\pi} 1+2\cos\theta+\cos^2\theta\,d\theta$$

$$= 2\int_0^{\pi} 1+2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\,d\theta$$

$$= 2\left(\theta+2\sin\theta+\frac{1}{2}\left(\theta+\frac{\sin 2\theta}{2}\right)\right)\Big|_0^{\pi}$$

$$= \left(\pi+\underbrace{2\sin\pi}_{0}+\frac{1}{2}\left(\pi+\underbrace{\frac{\sin 2\pi}{2}}_{0}\right)\right)-0$$

$$= 2\left(\frac{3\pi}{2}\right) = 3\pi$$

(8)

$$\int_1^2 \left[\int_x^{x^2} x y \, dy \right] dx$$

$$= \int_1^2 x \frac{y^2}{2} \Big|_x^{x^2} dx$$

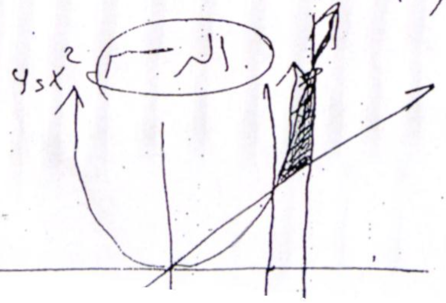
$$= \frac{1}{2} \int_1^2 x \left(x^4 - x^2 \right) dx$$

$$= \frac{1}{2} \int_1^2 x^5 - x^3 \, dx$$

$$= \frac{1}{2} \left[\frac{x^6}{6} - \frac{x^4}{4} \right]_1^2$$

$$= \frac{1}{2} \left[\left(\frac{2^6}{6} - \frac{2^4}{4} \right) - \left(\frac{1}{6} - \frac{1}{4} \right) \right]$$

$=$ ______



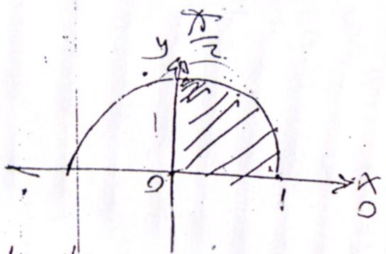
خاصة

(9)

(a) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} \dots$

$\therefore \tan^{-1} 0.2 = 0.2 + \frac{(0.2)^3}{3} - \frac{(0.2)^5}{5} \dots$

(b) $\int_0^1 \int_0^{\sqrt{1-x^2}} \sin(x^2+y^2)\, dy\, dx$ $\quad r^2 \quad r\,d\,r\,d\theta$



$\int_0^{\frac{\pi}{2}} \int_0^1 \sin r^2 \; r\,dr\,d\theta$

$\frac{1}{2} \int_0^{\frac{\pi}{2}} \int_0^1 \sin r^2 \; 2r \; r\,dr\,d\theta \Longrightarrow$

$\cancel{\int_0^{\frac{\pi}{2}} -\cos r^2 \Big|_0^1 d\theta = \frac{1}{2} \int_0^{\frac{\pi}{2}} -\cos 1 + \cos 0 \, d\theta}$

$\cancel{\frac{1}{2}(-\cos 1) \int_0^{\frac{\pi}{2}} d\theta = \frac{1}{2} (1-\cos 1)(\frac{\pi}{2})}$

HIGER TECHNOLOGICAL INSTITUTE
Tenth Of Ramadan City
Department Of Basic Science
Final Written Examination

Subject: MTH C (MTH 101)
Examiner: Board of Examiners

Term: Sept 2007 -Jan 2008
Time: 90 minutes

Answer three questions only from the following:

1-a) (i) Show that the series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p>1$, is convergent.

(ii) Test the series: $\sum_{n=1}^{\infty} \frac{2n+3}{(n+2)^2}$.

b) Use the double integration to find the area of the region bounded by the curves: $x=y^2$ and $x-y=2$.

c) If $z=\tan^{-1}(xy)$, $x=\tan t$, $y=t^{-1}$, find $\frac{dz}{dt}$.

2-a) Evaluate: $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} (1+x+y+z)^{-2}\, dz\, dx\, dy$.

b) If $f(x,y)=e^{xy}$, $\vec{u}=\vec{i}+2\vec{j}$, find the directional derivative: $D_u f(-1,3)$.

c) Find the Maclaurin series of the function: $f(x)=e^x$.

Approximate: $\int_0^{0.1} e^{x^2}\, dx$.

3-a) Let the equation of the curve C is given by
$x=t^3-3t$, $y=t^2-5t-1$, $t\in R$ .Find the slope of the tangent line to C at the point corresponding to $t=2$. Also find the point on C at which the tangent line is horizontal.

b)Find the area of the region bounded by the cardioid $r=2+2\cos\theta$

c) Evaluate: $\int_2^4 \int_0^x e^{x}\, dy\, dx$.

4- a) (i) Test the series: $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^2}$. (ii) Find the radius and interval of convergence of the power series: $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$.

b) Find the length of the cardioid: $r=1+\cos\theta$.

c) Find all the first partial derivatives of the function: $f(x,y,z)=(\ln x)\cos yz$.

$\cos y . z$

Sep 2001

(1) $\sum \frac{1}{n^p}$, $p>1$

الحل

$$\int_1^\infty \frac{1}{x^p}\,dx = \int_1^\infty x^{-p}\,dx$$

$$= \frac{x^{1-p}}{1-p}\Big|_1^\infty$$

at $p>1$

$\therefore 1-p \to -ve$

$-ve \leftarrow (1-p)$

$$\Rightarrow \cancel{\frac{\infty^{1-p}}{1-p}}_{\,0} - \frac{1^{1-p}}{1-p} = \frac{-1}{1-p}$$

Conv

note $p<1$ $\quad 1-p \Rightarrow +ve$

$\infty^{+v} = \infty \longrightarrow$ div

$$\sum \frac{2n+3}{(n+2)^2}$$

الحل

$$a_n = \frac{2n+3}{(n+2)^2} \longrightarrow b_n = \frac{n}{n^2} = \frac{1}{n}$$

$$\therefore \sum \frac{1}{n^p} \quad \underline{p=1} \quad \text{div}$$

$$\lim_{n\to\infty} \frac{2n+3}{(n+2)^2} * \frac{n}{1} = \frac{2}{1} = 2$$

$$\therefore b_n \longrightarrow \text{div}$$

$$\therefore a_n \longrightarrow \text{div}$$

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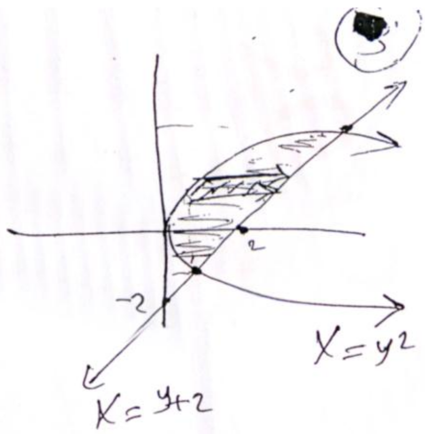
note لو نفس السؤال بس رابط

$$\sum (-1)^n \frac{2n+3}{(n+2)^2}$$ وطلب حدد نوع التقارب

$$\longrightarrow \sum (-1)^n \frac{2n+3}{(n+2)^2} \quad \text{conditional}$$

$$A = \int\int dx\,dy$$



أولاً نوجد حدود التكامل بحل المعادلتين

معاً جبرياً

$$x = y^2 \quad , \quad x - y = 2$$

$$\therefore y^2 - y - 2 = 0$$

$$(y+1)(y-2) = 0$$

$$y = -1 \qquad y = 2$$

$$\int_{-1}^{2}\int_{y^2}^{y+2} dx\,dy = \int_{-1}^{2} x \Big|_{y^2}^{y+2} dy$$

$$= \int_{-1}^{2} (y+2) - y^2\, dy$$

$$= \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_{-1}^{2}$$

(4)

(A) $z = \tan^{-1}(xy)$, $x = \tan t$
$y = t^{-1}$

find $\frac{dz}{dt}$

الحل

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$= \frac{1}{1+(xy)^2} \cdot y \cdot (\sec^2 t)$$

$$+ \frac{1}{1+(xy)^2} \cdot x \cdot (-t^{-2})$$

يجب نعوض عن y, x بدلالة t

$$\Rightarrow \frac{1}{1+\left(\frac{\tan t}{t}\right)^2} \cdot \frac{1}{t} \cdot (\sec^2 t)$$

$$+ \frac{1}{1+\left(\frac{\tan t}{t}\right)^2} \cdot \tan t \, (-t^{-2})$$

$f = e^{xy}$ $\quad \vec{u} = (1,2)$

$P(-1,3)$

الحل

(x,y)

$(-1,3)$

$f_x = e^{xy}\, y \longrightarrow f_x = 3e^{-3}$

$f_y = e^{xy}\, x \longrightarrow f_y = -e^{-3}$

$\vec{u} = (1,2) \rightarrow \hat{u} = \dfrac{(1,2)}{\sqrt{(1)^2+(2)^2}}$

$= \dfrac{(1,2)}{\sqrt{5}} = \left(\dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}}\right) = (a,b)$

$D_u = a f_x + b f_y$

$= \dfrac{1}{\sqrt{5}}(3e^{-3}) + \dfrac{2}{\sqrt{5}}(-e^{-3})$

(7) e^{x}

$f(x) = e^{x}$ $\qquad$ $f(0) = 1$

$f'(x) = e^{x}$ $\qquad$ $f'(0) = 1$

$f''(x) = e^{x}$ $\qquad$ $f''(0) = 1$

$\because f(x) = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \dots$

$$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} - \dots$$

to approximate $\int_0^{0.1} e^{x^2} dx$

نعوض x^2

$$\therefore e^{x^2} = 1 + \frac{x^2}{1!} + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} - \dots$$

$$= 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!}$$

$$\therefore \int_0^{0.1} e^{x^2} dx = \int_0^{0.1} \text{———} \, dx$$

$$= x + \frac{x^3}{3(1!)} + \frac{x^5}{5(2!)} + \frac{x^7}{7(3!)} - \dots \Big|_0^{0.1}$$

صح

(A) $x = t^3 - 3t \ , \ y = t^2 - 5t - 1$ (3 علامات تم الورقة) 7

الحل

$\dfrac{dy}{dt} = 2t - 5 \qquad \dfrac{dx}{dt} = 3t^2 - 3$

$m = \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{2t-5}{3t^2-3}$

at $t = 2$ $\quad m = \dfrac{-1}{9}$

to be horizontal

$\therefore y' = 0$

$\dfrac{2t-5}{3t^2-3} = 0 \qquad 2t - 5 = 0$

$t = \dfrac{5}{2}$

$x = \left(\dfrac{5}{2}\right)^3 - 3\left(\dfrac{5}{2}\right) = \_\_$

$y = \left(\dfrac{5}{2}\right)^2 - 5\left(\dfrac{5}{2}\right) - 1 = \_\_$

Ⓐ $r = 2(1+\cos\theta)$ ⑧

دحول نبر العوبد
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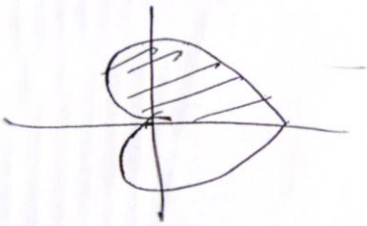
$$A = \cancel{2}\int_0^{\pi} \cancel{\tfrac{1}{2}}\, r^2\, d\theta$$



$$= \int_0^{\pi} \left(2(1+\cos\theta)\right)^2 d\theta$$

$$= 4\int_0^{\pi} 1 + 2\cos\theta + \cos^2\theta \; d\theta$$

$$= 4\int_0^{\pi} 1 + 2\cos\theta + \tfrac{1}{2}(1+\cos 2\theta)\, d\theta$$

$$= 4\left(\theta + 2\sin\theta + \tfrac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right)\right)\Big|_0^{\pi}$$

$$= 4\left[\left(\pi + 2\cancel{\sin\pi}_{\,0} + \tfrac{1}{2}\left(\pi + \cancel{\frac{\sin 2\pi}{2}}_{\,0}\right)\right) - \right.$$

$$= 4\left(\frac{3\pi}{2}\right) = 6\pi$$

$$\int_2^4 \int_0^x e^x \, dy \, dx$$

$$\int_2^4 y e^x \Big|_0^x \, dx$$

$$\int u\,dv = uv - \int v\,du$$

$u = x \qquad dv = e^x$

$du = dx \qquad v = e^x$

$$= \int_2^4 x e^x \, dx \longrightarrow \text{By parts}$$

$$= x e^x - \int e^x \, dx \Big|_2^4$$

$$= x e^x - e^x \Big|_2^4$$

$$= (4e^4 - e^4) - (2e^2 - e^2)$$

$$= 3e^4 - e^2$$

test $\sum_{n=1} (-1)^{n-1} \frac{1}{n^2}$

① $\lim_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty} = 0$

② $f(x) = \frac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-3}$

$f'(1) = -2 < 0$

conv

$\because \sum \frac{1}{n^2}$ $p = 2 > 1$ conv

$\because \sum (-1)^{n-1} \frac{1}{n^2}$

absolute conv

$\textcircled{4}$ $\sum \frac{x^n}{\sqrt{n}}$ power series

الحل

$a_n = \frac{x^n}{\sqrt{n}}$ $a_{n+1} = \frac{x^{n+1}}{\sqrt{n+1}}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{x^n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{x^n} \cdot \frac{\sqrt{n}}{\sqrt{n+1}} \right| < 1$$

$\to 1$

$$|x| < 1$$

$$-1 < x < 1$$

to test end points

at $X = -1$ عوض بأطراف الفترة

$$\sum (-1)^n \cdot \frac{1}{\sqrt{n}}$$

① $\lim_{n \to \infty} \frac{1}{\sqrt{n}} = \frac{1}{\infty} = 0$

② $f(x) = \frac{1}{\sqrt{x}} = X^{-\frac{1}{2}}$

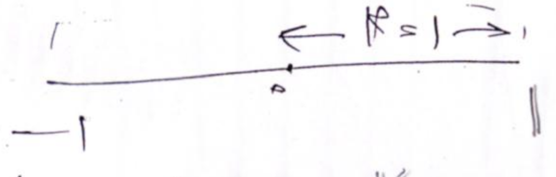
$f'(x) = -\frac{1}{2} X^{-\frac{3}{2}}$

$f'(1) = -\frac{1}{2} < 0$ conv

at $X = 1$

$$\sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{\frac{1}{2}}} \quad p = \frac{1}{2} < 1 \quad \text{div}$$

interval $-1 \leq X < 1$



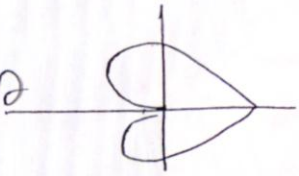
Radius = 1 $\quad X_0 = 0$

length جد طول المنحنى

$r = 1+\cos\theta$

$\frac{dr}{d\theta} = -\sin\theta$

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$



$$\Rightarrow L = 2 \int_0^{\pi} \sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\, d\theta$$

FG

$$2 \int_0^{\pi} \sqrt{1 + 2\cos\theta + \underbrace{\cos^2\theta + \sin^2\theta}_{1}}\, d\theta$$

$$2 \int_0^{\pi} \sqrt{2 + 2\cos\theta}\, d\theta$$

$$2 \int_0^{\pi} \sqrt{2(\underbrace{1+\cos\theta}_{2\cos^2\frac{\theta}{2}})}\, d\theta$$

$$= 2 \int_0^{\pi} \sqrt{2\left(2\cos^2\frac{\theta}{2}\right)}\, d\theta = 4 \int_0^{\pi} \cos\frac{\theta}{2}\, d\theta$$

$$\therefore = 4 \left(\frac{\sin\frac{\theta}{2}}{\frac{1}{2}} \right)\Bigg|_0^{\pi} = 8\left(\sin\frac{\pi}{2} - \sin 0\right) = 8$$

Higher Technological Institute

10^{th} of Ramadan City

Department of Basic Sciences

Subject: Mathematics (C) (Math 101) Term: June/August 2010

Examiner: Examination committee Time: 90 minutes

Final Written Examination

Answer the following questions:

Question (1) [10Marks]

(a) Use the limit comparison test to discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{4n^3 - 8n}}$. (4Marks)

(b) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-3)^n}{n}$ (6Marks)

Question (2) [10Marks]

(a) Use Maclaurin series of the functions $e^{i\theta}$, $\sin\theta$, $\cos\theta$ to verify that

$e^{i\theta} = \cos\theta + i\sin\theta$, where $i = \sqrt{-1}$. (5Marks)

(b) Find the length of the curve: $x = e^t \cos t$, $y = e^t \sin t$, $0 \le t \le 2\pi$. (5Marks)

$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Question (3) [10Marks]

(a) Find the area of the region bounded by the graph of $r = 2\sin(2\theta)$, $0 \le \theta \le 2\pi$. (5Marks)

$A = 2\pi \int r^2\, dr$

(b) Find equations of tangent plane and normal line at the point P (- 3 , 0 , 8/5) to the surface

$36x^2 + 100y^2 + 225z^2 = 900$. $\frac{y-y_0}{x-x_0} = m$ $\frac{y-y_0}{x-x_0} = -\frac{1}{m}$ (5Marks)

Question (4) [10Marks]

(a) Find the maximum, minimum values and saddle points of the function

$f(x,y) = \frac{1}{3}x^3 + \frac{1}{3}y^3 - \frac{3}{2}x^2 - 4y$. (5Marks)

(b) Evaluate the following double integral $\int_0^{0.09}\int_{\sqrt{y}}^{0.3} \sin x^3\, dx\, dy$. (5Marks)

Good luck

Final June-August 2010 (1)

(1) $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{4n^3-8n}}$$

الحل

$$a_n = \frac{1}{\sqrt{4n^3-8n}} \qquad b_n = \frac{1}{n^{3/2}}$$

$$\because \sum \frac{1}{n^{3/2}} \qquad p = \frac{3}{2} > 1 \qquad \text{Conv}$$

$$\because \lim_{n \to \infty} \frac{a_n}{b_n}$$

$$= \lim_{n \to \infty} \frac{1}{\sqrt{4n^3-8n}} * \frac{n^{3/2}}{1} = \frac{1}{2} \to$$

$$\because b_n \longrightarrow \text{conv}$$

$$\therefore a_n \longrightarrow \text{conv}$$

$$\rightarrow \cancel{2}\int_0^{2\pi} \frac{1}{\cancel{2}}(1-\cos 4\theta)\,d\theta \qquad 4$$

$$= \theta - \frac{\sin 4\theta}{4}\Big|_0^{2\pi}$$

$$= \left(2\pi - \frac{\cancel{\sin 8\pi}}{4}\right) - \left(0 - \frac{\cancel{\sin 0}}{4}\right)$$

$$= \boxed{2\pi}$$

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b) $36x^2+100y^2+225z^2=900$

$P\left(-3, 0, \frac{8}{5}\right)$

الحل

$f = 36x^2+100y^2+225z^2-900 = 0$

$\left(-3, 0, \frac{8}{5}\right)$

$f_x = 72x$	$f_x = -216$
$f_y = 200y$	$f_y = 0$
$f_z = 450z$	$f_z = 720$

Tangent plane

(5)

$$(x-x_0)f_x+(y-y_0)f_y+(z-z_0)f_z=0$$

$$(x+3)(-216)+(y-0)(0)+(z-\frac{8}{5})(720)=0$$

Normal line

$$\frac{x-x_0}{f_x}=\frac{y-y_0}{f_y}=\frac{z-z_0}{f_z}$$

$$\Rightarrow \frac{x+3}{-216}=\frac{y-0}{0}=\frac{z-\frac{8}{5}}{720}$$

الطلاب الاحرار

Q(4) $f = \frac{1}{3}x^3 + \frac{1}{3}y^3 - \frac{3}{2}x^2 - 4y$ (6)

الحل

$f_x = x^2 - 3x = 0 \longrightarrow (1)$

$f_y = y^2 - 4 = 0 \longrightarrow (2)$

from (1) $x(x-3) = 0$ $x = 0$ $x = 3$

from (2) $y^2 = 4$ $y = \pm 2$

points $(0,2)$, $(0,-2)$

$(3,2)$, $(3,-2)$

Then we find $F = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$F = (2x-3) \cdot 2y - (0)^2$

$\boxed{F = 2y(2x-3)}$