Water Hammer in Piping Systems

Lecture Notes

Prepared by

Dr. Aly El-Bahrawy, Professor
Irrigation and Hydraulics Department
Faculty of Engineering, Ain Shams University

March,

Table of contents

Introduction

Unsteady flow in piping systems is a common occurrence. Indeed, steady flow is so rare that one might question the advisability of devoting so much time to a study of its behavior. Virtually all hydraulic design is based on steady flow analysis and, in many cases, the unsteadiness occurring in a pipeline system is of little consequence because of its transient nature and its small magnitude of change. It is with those few cases wherein significant changes in velocity cause large changes in pressure that we are concerned.

Unsteady flows are divided into two categories, depending on the type of analysis required to accurately describe the flow behavior. The first, called surge or rigid water column theory treats the fluid as an inelastic substance wherein pressure changes propagate instantaneously throughout the system and elastic properties of the pipe walls are of no consequence. The equations describing this type of flow are generally ordinary differential equations which can be solved in close form or with relatively straightforward numerical techniques. Where applicable, this approach is the easiest to apply and should always be considered as a possibility to adequately approximate problems under considerations. The second category of problems is classified under elastic or water hammer theory wherein the elasticity of both the fluid and the pipe walls is taken into account in the calculations. Pressure waves created by velocity changes depend on these elastic properties and they propagate throughout the pipeline system at speeds depending directly on these elastic properties. Often it is not clear which type of analysis should be used because there is no distinct line of demarcation between the two areas of application. On the other hand, there are cases where it is obvious which type of approach should be used. Fore example, if a large storage tank 50 feet in diameter and 75 feet tall were to be drained through a 6-inch pipeline 1000 feet long, it would be foolish to use elastic theory in a traditional water hammer analysis. Yet, if during the draining process, there was the possibility of having to close the discharge valve suddenly, then significant hammer occur and elastic theory should used.

The Unsteady Flow Equations

To analyze unsteady flow problems in pipe systems, we must begin with developing an equation describing flow in a single pipe. The approach is to apply Newton's second law to a small cylindrical fluid particle at the pipe centerline. The resulting differential equation of unsteady flow is known as the Euler equation and it applies to one-dimensional flow in the pipe. Two or three-dimensional flows are not of significance in this situation, hence the one-dimensional analysis is adequate. Further, because of the differential size of the fluid element, the resulting differential equation is equally valid for compressible and incompressible flow and can be used in both rigid water column and elastic analyses.

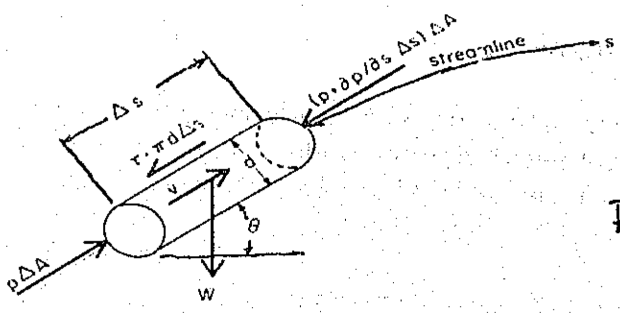


Fig 1

Considering only the stream line direction, Newton's second law gives,

$$\sum F_s = m\,a_s = m\frac{dv}{dt}$$

Where m is the fluid particle mass, and s signifies the streamline direction. Substituting the force components and mass from the figure in the equation results in,

$$p\Delta A - \left(p + \frac{\partial p}{\partial s}\Delta s\right)\Delta A - W\sin\theta - \tau\Delta s\pi d = \frac{W}{g}\frac{dv}{dt}$$

After some manipulation, we end up with the one-dimensional Euler equation,

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{4\tau}{\gamma d} = \frac{1}{g}\frac{dv}{dt}$$

Expanding the particle diameter to the size of the pipe cross-section and introducing the average velocity V gives a more useful equation;

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{4\tau_o}{\gamma D} = \frac{1}{g}\frac{dV}{dt}$$

Where D is the pipe diameter and τ_o is the shear stress at the wall. Using the relation between τ_o and f of the Darcy-Weisbach equation in the form,

$$\tau_o = \frac{1}{8}\rho\, f\, V|V|$$

The derived equation becomes,

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{f}{D}\frac{V^2}{2g} = \frac{1}{g}\frac{dV}{dt}$$

Knowing that

$$h = z + \frac{p}{\gamma}$$

The equation can read

$$-\frac{\partial h}{\partial s} - \frac{f}{D}\frac{V^2}{2g} = \frac{1}{g}\frac{dV}{dt}$$

Rigid Water Column Theory

The unsteady flow equation can be used to solve a wide range of pipeline problems which fall within the domain of rigid water column theory. We will begin with some of the simple problems and proceed to more comprehensive ones.

Flow Establishment in a Horizontal Pipe

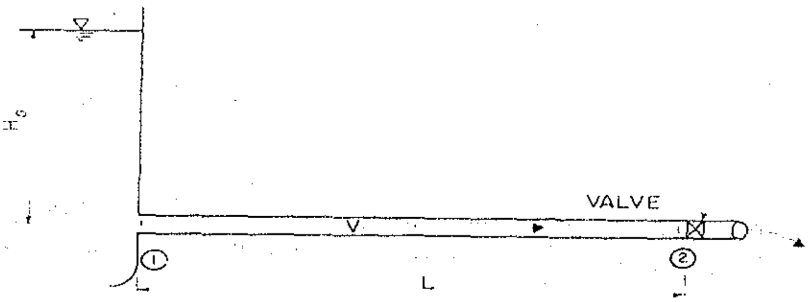


Fig 2

Simple System for Applying Rigid Water Column Theory

If the discharge in the pipeline shown in Figure is controlled by the valve at the downstream end, the pressure in the pipe is everywhere equal to H_o when the valve is closed. When the valve is suddenly opened, the pressure at the valve drops instantly to zero and the fluid begins to accelerate. The equation describing this flow is obtained by integrating the unsteady flow equation with respect to s,

$$-\int_L \frac{\partial h}{\partial s}\,ds - \int_L \frac{fV^2}{2gD}\,ds = \int_L \frac{1}{g}\frac{dV}{dt}\,ds$$

In a horizontal constant diameter pipe, the integration is made assuming that f in unsteady flow is the same as for a steady flow at a velocity equal to the instantaneous value. The result is,

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - \frac{fL}{2gD}V^2 = \frac{L}{g}\frac{dV}{dt}$$

The pressure head at point 1 is H_o and at point 2 is zero, and the equation reduces to,

$$H_o - \frac{fL}{2gD}V^2 = \frac{L}{g}\frac{dV}{dt}$$

Separating the variables and integrating we get,

$$\int dt = \frac{L}{g} \int \frac{dV}{H_o - \frac{fL}{2gD} V^2}$$

The integration gives the following equation for the time necessary to accelerate the flow to a given velocity V,

$$t = \sqrt{\frac{LD}{2gfH_o}} \ln \frac{\sqrt{\frac{2gDH_o}{fL}} + V}{\sqrt{\frac{2gDH_o}{fL}} - V}$$

Where ~~log~~ ln denotes natural logarithm. Recognizing that square root in both the numerator and denominator is the steady state velocity V_o, then the equation for t becomes

$$t = \frac{LV_o}{2gH_o} \ln \frac{V_o + V}{V_o - V}$$

It is important to note that as steady flow is approached, V approaches V_0 and t approaches ∞, which is unacceptable, so we assume that when $V = 0.99\ V_o$, the steady flow is essentially achieved. With this interpretation,

$$t_{99} = 2.65 \frac{LV_o}{gH_o}$$

Example 1

A horizontal pipe 24 inch in diameter and 10000 foot long leaves a reservoir 100 ft below the surface and terminate in a valve. The steady state friction factor is 0.018 and it is assume d to remain constant during the acceleration process. If the valve opens suddenly, calculate how long it will take for the velocity to reach 99 percent of its final values. Neglect minor losses.

$$h_f = f \frac{L}{D} \frac{V_o^2}{2g} = \frac{0.018 \times 10000 \times V_o^2}{2 \times 2 \times 32.2} = 100\ m$$

To use the equation for the time derived above, V_o has to be calculated first,

$V_o = 8.46$ fps, and

$$t_{99} = 2.65 \frac{10000 \times 8.46}{32.2 \times 100} = 70 \text{ sec}$$

The following graph illustrates how the velocity approaches its steady state value with time.

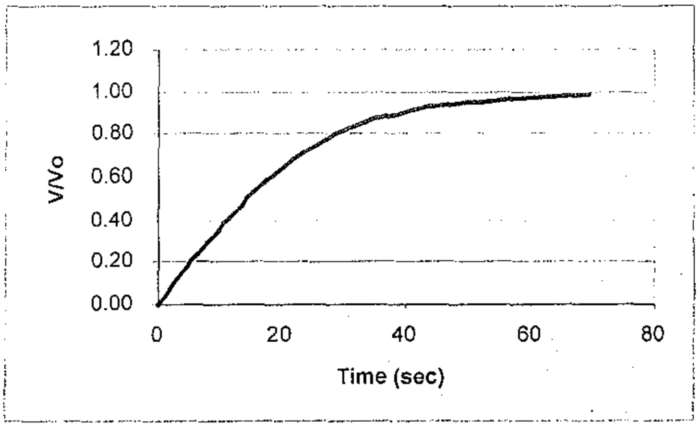


Fig 3

Pressure Caused by Valve Closure in a Horizontal Pipe

Valve closure can cause some analysis problems beyond those of instantaneous valve openings. The difficulty occurring in this problem is precipitated by the fact that the pressure just upstream of the valve is no longer zero, but is determined by loss characteristics of the flow through the valve.

The differential equation representing this problem is the same as before,

$$h_1 - h_2 - \frac{fL}{2gD}V^2 = \frac{L}{g}\frac{dV}{dt}$$

Unfortunately, there are two independent variables, p_2 and V, and another equation is needed (boundary condition), which is typical of unsteady flow problems.

There is indeed a limit of applicability to this rigid column approach as discussed before, which can be seen with the above equation. As faster and faster valve closure times are used, dV/dt becomes quite large and, in the limit, goes to infinity. According to the equation, in the limit, h_2 approaches ∞ also. The point at which rigid water column theory fails to give acceptable results and a move to elastic theory is necessary is hard to establish, because it depends on the individual problem and the accuracy in analysis required.

Example 2

Water flows from one reservoir to another through the pipe at a velocity of 10 fps. The shutdown plan calls for a valve closure scheme which will cause the velocity to decrease linearly to zero in 100 sec. The valve is located at the center of a 6440 ft long pipeline. Estimate the maximum and minimum pressure which will occur in the system, locate them, and give the time at which they will occur.

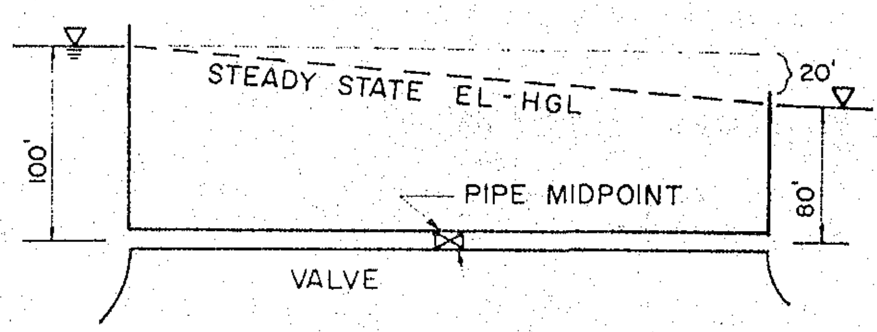


Fig 4

The general form of the unsteady flow equation applying to this situation is,

$$\frac{p_1}{\gamma}-\frac{p_2}{\gamma}-\frac{fL}{2gD}V^2=\frac{L}{g}\frac{dV}{dt}$$

Given that the velocity will decrease linearly with time,

$$\frac{dV}{dt}=\frac{-10}{100}=-0.10\ ft/s^2$$

The problem is solved by considering two sections, upstream of the valve.

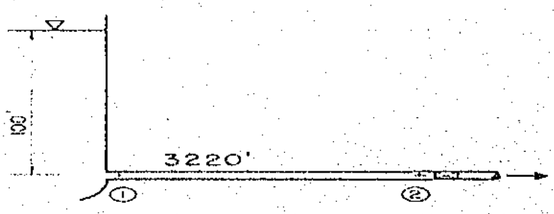


Fig 5

$$\frac{p_2}{\gamma}=100-\frac{fL}{2gD}V^2-\frac{3220}{32.2}(-0.10)=110-\frac{fL}{2gD}V^2$$

and downstream of the valve,

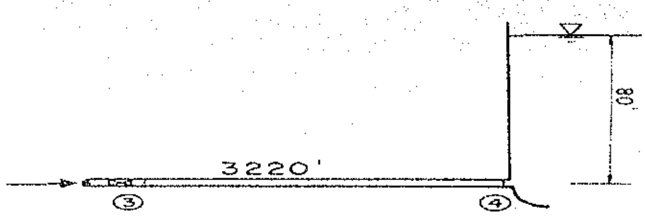


Fig 6

$$\frac{p_3}{\gamma}=80+\frac{fL}{2gD}V^2+\frac{3220}{32.2}(-0.10)=70+\frac{fL}{2gD}V^2$$

The downstream max pressure is 110 ft at t = 100 sec, and the min is 90 ft at t = 0, while the upstream max pressure is 90 ft at t = 0, and the min is 70 ft at t = 100 sec.

Simplified Analysis of Water Hammer

Water hammer phenomenon can be simply described as the 'inertia pressure' resulting from either acceleration of deceleration of a water column.

Inertia pressure is the difference in piezometric pressure, simply

Force = mass x acceleration

$$-\Delta p\, A = \rho\, A\, L \frac{\partial v}{\partial t}$$

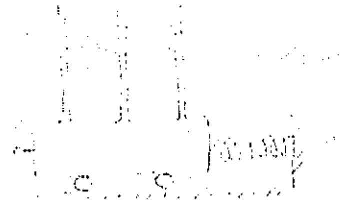


Fig 7

or

$$\Delta H = \frac{-L}{g}\frac{dv}{dt}$$

which is the dynamic equation for uniform cross section and incompressible fluid.

The same result can be obtained from the detailed analysis above by neglecting the friction losses, where the unsteady equation takes the form,

$$\frac{\partial h}{\partial x} + \frac{1}{g}\frac{\partial V}{\partial t} = 0$$

The previous analysis assumes that at any instant in time V is constant along the length of the pipe, i.e. 'rigid' or 'inelastic' water column, or

$$V = \frac{Q}{A} \quad for\ all\ x$$

The above is true if the closure is gradual.

Example 3

Assuming water is discharged from a reservoir through a pipe of length 1500 m, where the steady state velocity is 2.3 m/s, and the head over the pipe outlet is 120 m. If a valve at the downstream end of the pipe is closed so that the velocity decreases to 1.2 m/s in 20 seconds. Calculate the max increase in pressure using the simplified equation above.

Assuming the closure is uniform, i.e.

$$\frac{\partial V}{\partial t} = const = \frac{\Delta V}{\Delta t}$$

$$\Delta H = -\frac{L}{g}\frac{\Delta V}{\Delta t} = -\frac{L}{g}\frac{(V_f - V_o)}{\Delta t} = -\frac{1500}{9.81}\frac{(1.2 - 2.3)}{20} = 8.41\, m$$

In the previous analysis, it was assumed that the change in velocity with time is linear. However, it is more realistic to assume that the valve area decreases linearly with time. By doing this the change in velocity with time will not be linear any more and the problem will be solved by dividing the closure time into smaller steps. The following two examples show how the realistic closure assumption is used to solve the problem.

Example 4

As the previous example but assume that valve area is closed from 100% to 55% in 20 sec and the rate of closure is such that

$$\frac{\partial \alpha}{\partial t} = const$$

Let

$$\alpha = \frac{a}{a_o}$$

Where a is the effective valve area, and a_o is the full open area.

Assuming,

$$Q = AV = a\sqrt{2g(H_o + \Delta H)} \quad \text{and } Q_o = AV_o = a_o\sqrt{2gH_o}$$

where ΔH is the water hammer inertia head, then

$$\frac{V}{V_o} = \frac{a}{a_o}\sqrt{\frac{H_o + \Delta H}{H_o}} \qquad \text{or}$$

$$V = \alpha V_o \sqrt{\frac{H_o + \Delta H}{H_0}} \qquad \text{the boundary condition}$$

V and ΔH are calculated by solving the above equation with the simplified unsteady flow equation

$$\Delta H = -\frac{L}{g}\frac{\Delta V}{\Delta t} = -\frac{L}{g}\frac{(V_f - V_o)}{\Delta t}$$

If the closure is similar to the previous example, then solving the two equations together we get,

$$V_f = 0.55 \, x \, 2.3 \sqrt{\frac{120 + \Delta H}{120}} = 0.115\sqrt{120 + \Delta H}$$

$$\Delta H = -\frac{1500}{9.81}\frac{(V_f - 2.3)}{20} = 7.645\,(2.3 - V_f)$$

Squaring the first equation and substituting for ΔH from the second equation the following quadratic equation results,

$$V_f^2 + 0.101\,V_f - 1.8196 = 0$$

The solution of the above equation is,

V_f = 1.305 m/s   and ΔH = 7.61 m

If the closure is linear from 100% to 45% then V_f = 1.0746 and ΔH = 9.368 m.

Example 5

Same as the previous example but the history of the ratio $\frac{da}{dt}$ or $\frac{d\alpha}{dt}$ with time is known. As an example, α may reduce linearly from 1.0 to 0.45.

The solution method is to divide closure into several small increments within which the $\frac{\partial V}{\partial t}$ ratio may be reasonably assumed constant.

The function α (t) may itself be nonlinear. For each increment use appropriate Δt (need not be constant) and solve for ΔV and ΔH. Use the largest ΔH as the answer to the problem. The following is the spreadsheet solution to the problem.

To facilitate hand calculations for each time increment the following solution to the quadratic equation is developed for the ith interval,

$$V_i = \alpha_i\ V_o \sqrt{\frac{H_o + \Delta H_i}{H_0}}$$

$$\Delta H_i = -\frac{L}{g}\frac{(V_i - V_{i-1})}{(t_i - t_{i-1})}$$

$$V_i^2 + \alpha_i^2 V_o^2 + \left(\frac{\alpha_i^2 V_o^2}{H_o}\right)\Delta H_i$$

$$\therefore V_i^2 = \left(\frac{\alpha_i^2 V_o^2}{H_o}\right)\frac{L}{g}\frac{V_i}{\Delta t} - \left(\frac{\alpha_i^2 V_o^2}{H_o}\right)\frac{L}{g}\frac{V_{i-1}}{\Delta t} - \alpha_i^2 V_o^2 = 0$$

$$\therefore V_i = -\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + c}$$

where

$$b = \left(\frac{\alpha_i^2 V_o^2}{H_o}\right)\frac{L}{g\Delta t}$$

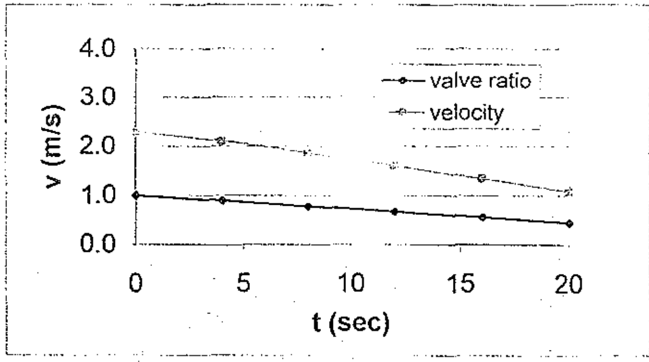
and

$$c = bV_{i-1} + \alpha_i^2 V_o^2$$

Water Hammer (Rigid Water Column)

Input Data

L	1500	m	length of pipe
Vo	2.3	m/s	intial velocity
Ho	120	m	initial head
tc	20	sec	closure time
α_o	1		intial valve closure ratio
α_c	0.45		final valve closure ratio
Δt	4	sec	time increment
g	9.81	m/s2	acceleration of gravity
n	5		number of time increments



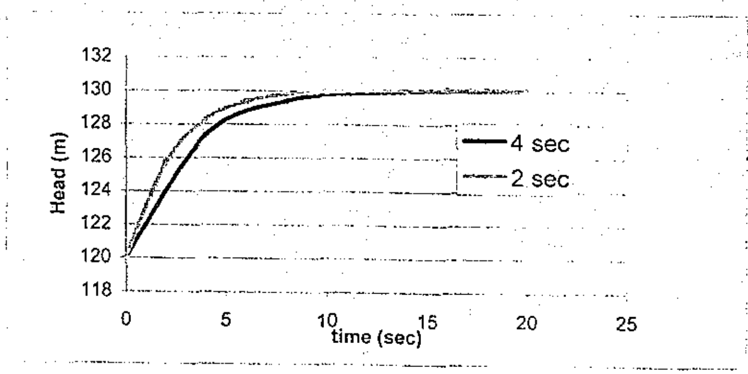
Intermediate Calculations

t	α	b	c	V	ΔH	H
0	1.000			2.30		120
4	0.890	1.33	7.26	2.11	7.32	127.32
8	0.780	1.03	5.38	1.86	9.39	129.39
12	0.670	0.76	3.78	1.60	9.92	129.92
16	0.560	0.53	2.51	1.34	10.04	130.04
20	0.450	0.34	1.53	1.08	10.06	130.06

Intermediate Calculations (automatic)

t	α	V	ΔH	H
0	1.000	2.30	0.00	120.00
4	0.890	2.11	7.32	127.32
8	0.780	1.86	9.39	129.39
12	0.670	1.60	9.92	129.92
16	0.560	1.34	10.04	130.04
20	0.450	1.08	10.06	130.06

t	α	V	ΔH	H
0	1.000	2.3	0	120
2	0.945	2.225	5.74	125.74
4	0.890	2.117	8.29	128.29
6	0.835	1.994	9.36	129.36
8	0.780	1.866	9.80	129.80
10	0.725	1.735	9.97	129.97
12	0.670	1.604	10.04	130.04
14	0.615	1.473	10.06	130.06
16	0.560	1.341	10.07	130.07
18	0.505	1.209	10.07	130.07
20	0.450	1.078	10.07	130.07



Elastic Water Column

The dynamic equation

$\Delta H = -\frac{L}{g}\frac{\Delta V}{\Delta t}$ *suggests that* $\Delta H \to \infty$ *as* $\Delta t \to 0$

When closure is very rapid, the elasticity of the water column and the containing pipe result in creation of a pressure wave, i.e.

$\frac{\partial V}{\partial x} \neq 0$ (cf. traffic approaching stop light !)

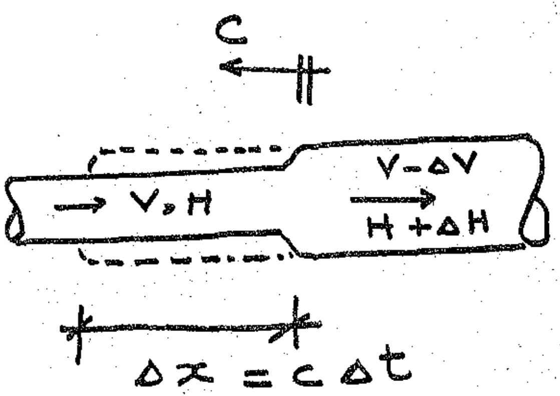


Fig. 8

Consider control volume as water which experiences change in momentum in time Δt.

Force = change of momentum / second

$$\rho g \Delta H A = -\frac{d}{dt}(m\Delta V) = -\frac{dm}{dx}\frac{dx}{dt}\Delta V = -(\rho A)c\Delta V$$

Or

$\Delta H = -\frac{c}{g}\Delta V$ Which is the dynamic equation for elastic water column (independent of Δt)

Example 6

If c = 1000 m/s and velocity is abruptly reduced from 2.3 to 1.2 m/s find ΔH.

$$\Delta H = -\frac{c}{g}\Delta V = -\frac{1000}{9.81}(1.2-2.3) = 112.1\ m$$

Simplified Description of Water Hammer

To grasp a basic understanding of the action of a pipe system carrying liquid under the action of water hammer waves, it is easiest to consider as simple a system as possible. The system we will examine is shown in Figure 13 below as horizontal, constant diameter pipe leading from a reservoir to some unknown destination far downstream. A valve is placed a distance L from the reservoir. Friction in the line is assumed negligible to simplify the analysis; and because velocity heads are generally quite small in relation to water hammer pressures, the difference between the energy grade line and the hydraulic grade line will be neglected. Water hammer will be introduced into the system by suddenly closing the valve. The activity will occur both upstream and downstream of the valve but for our purposes, we will observe only what occurs upstream of the valve.

- Upon sudden closure of the valve the velocity of water at the valve is forced suddenly to zero. As a consequence, the pressure head at the valve increases suddenly by an amount ΔH (see Figure 13). The magnitude of ΔH is just the amount of pressure head necessary to change the momentum of the liquid initially flowing at velocity V at the valve to zero. The increase in pressure at the valve results in a swelling of the pipe and an increase in the density of the liquid. The amount of pipe stretching and liquid density increase depends on the pipe material and size and the liquid elasticity. Generally, for common pipe materials and liquids, the percentage change is less than 0.5 percent. The deformation has been greatly exaggerated in Figure for purposes of illustration.

- The pressure increase propagates upstream at a wave speed of a, which is determined by the elastic properties of the system and the liquid and the system geometry. The wave speed will remain constant so long as the above remain constant.

- Traveling at a speed a, the wave will reach the reservoir in a time L/a. At this time the velocity in the pipe is everywhere zero, the pressure head is everywhere $H+\Delta H$, the pipe is stretched, and the fluid is compressed. Under these conditions the liquid in the pipe is under a condition of non-equilibrium because the pressure head in the reservoir is only H.

- As a result, flow begins to occur toward the reservoir as the distended pipe ejects liquid in that direction. The reverse velocity is equal in magnitude to the initial steady velocity (as a result of neglecting friction) and the source of liquid for the reverse flow is the liquid previously stored in the stretched pipe walls as compressed liquid.

- This process continues and at time 2 L/a, the pressure has returned to normal (but with reverse flow occurring) throughout the pipe.

- However, there is no source of liquid at the valve to supply the upstream flow hence the pressure head drops an additional ΔH to force the reverse velocity to zero. This drop in pressure causes the pipe to shrink and the liquid to expand.

- At time 3 L/a this effect has propagated to the reservoir and the velocity of flow is everywhere zero. However, the pipe pressure head is ΔH below that of the reservoir.

- Consequently, the pipe sucks in liquid from the reservoir creating a velocity of flow equal to and in the same direction as the original steady flow. While this is occurring the pressure in the pipe is also returning to its original value.

- After time 4 L/a this wave has reached the valve and at this instant the flow is identical to its original steady state configuration. This elapsed time constitutes one wave period. As time goes on, this cycle of events will continue without abatement (in the absence of friction).

Some fundamental concepts can be gained from examining more closely what occurs in this system. For example, it is clear that the time parameter which best describes the sequence of events in a meaningful fashion is not time alone but the ratio L/a. It is informative to plot the pressure head at various points in the pipeline as a function of time as shown in Figure 14. Note the pressure head at the valve fluctuates between $H \pm \Delta H$ whereas the pressure head at other locations also experiences periods of time when its value is H.

One basic point can be made from Figure (14-b). Note that the pressure does not increase at a point until enough time has occurred for the wave to travel from the closed valve. Once the pressure head has increased, it remains there only long enough for "relief' to arrive back from the reservoir. This idea of "time of communication" or "message propagation time" is fundamental to a good understanding of the happenings in a system undergoing water hammer.

A second important point can be seen by examining Figure (14-a) more closely. Suppose that instead of closing the valve suddenly, we were to close it in 10 steps, each increasing the pressure head at the valve by $\Delta H/10$. A further requirement would be that the complete closure of the valve would be accomplished before 2 L/a seconds had elapsed. It is clear that the pressure head at the valve would still build up to the full ΔH value because "relief" from the reservoir could not arrive before 2 L/a seconds. The point to be made is that a valve need not be closed suddenly to create the maximum water hammer pressure. Indeed, any closure time less than the time necessary for relief to return from a reservoir (a larger pipe may also act much like a reservoir) will result in full water hammer pressures. In fact, as we will see later, because of the manner in which a valve shuts off flow in a pipeline by creating large head losses, it may be necessary to close the valve in a time much greater than 2 L/a to prevent high pressures from occurring.

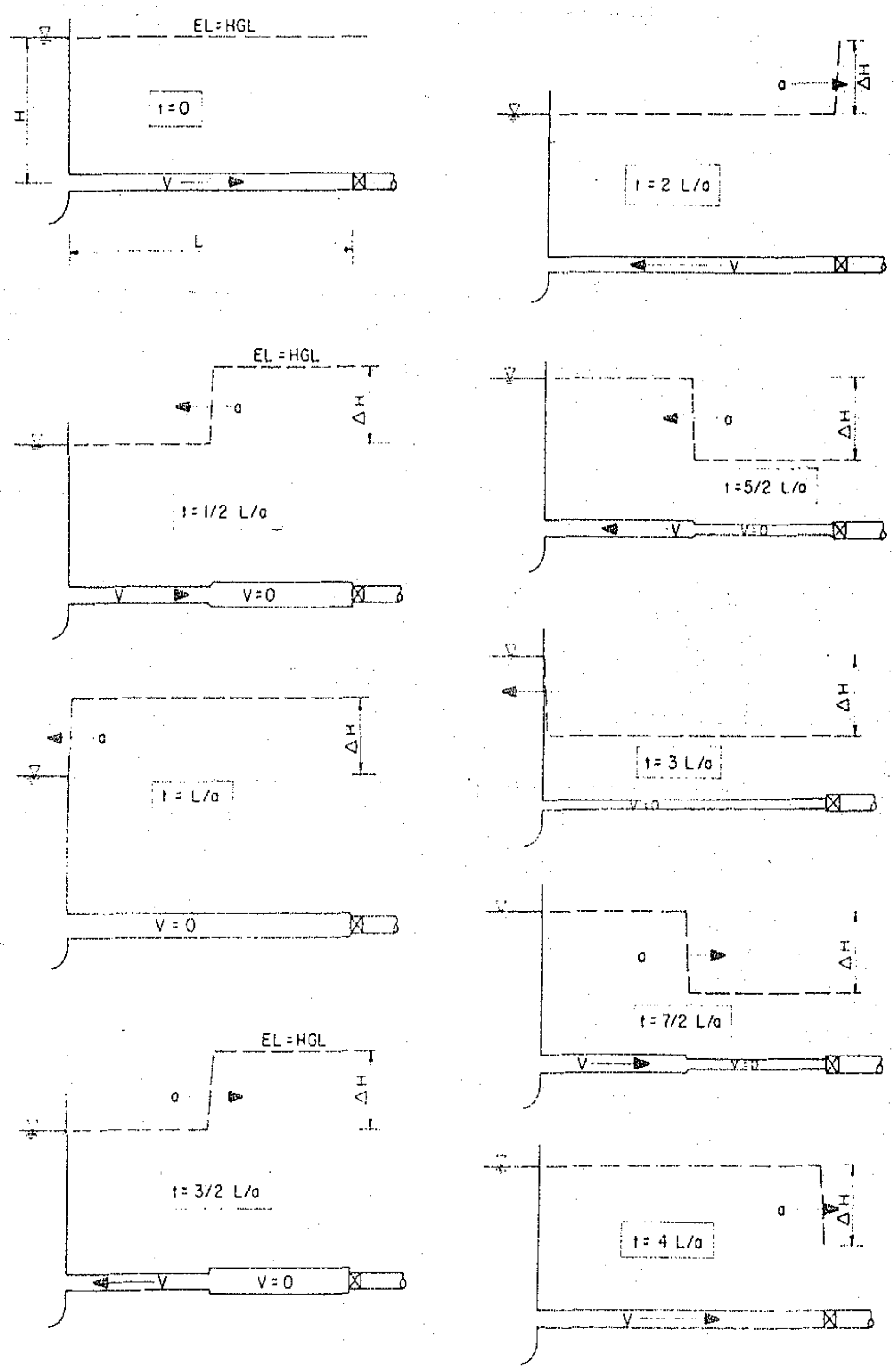
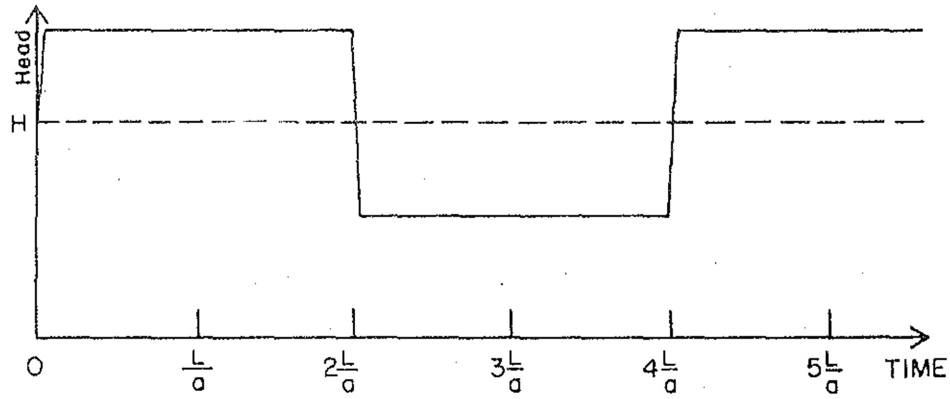
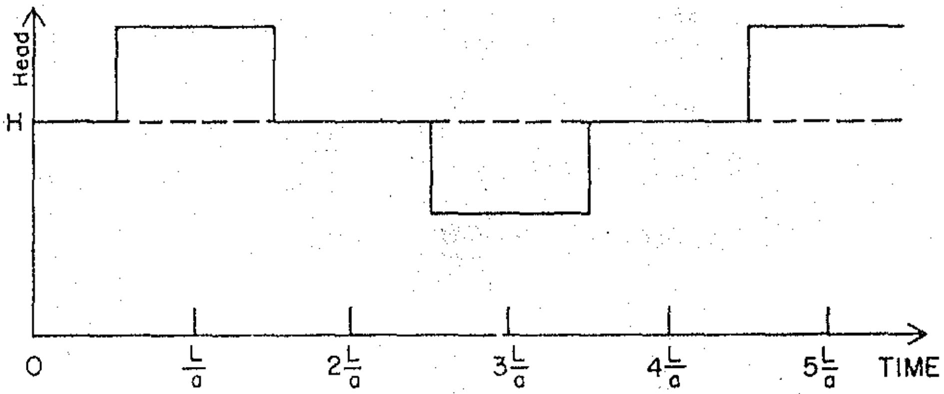

EL=HGL
t=0
H
V
L
t=2 L/a
a
ΔH
EL=HGL
t=1/2 L/a
V=0
t=5/2 L/a
t=L/a
V=0
t=3 L/a
EL=HGL
t=3/2 L/a
V=0
t=7/2 L/a
t=4 L/a

Fig 13

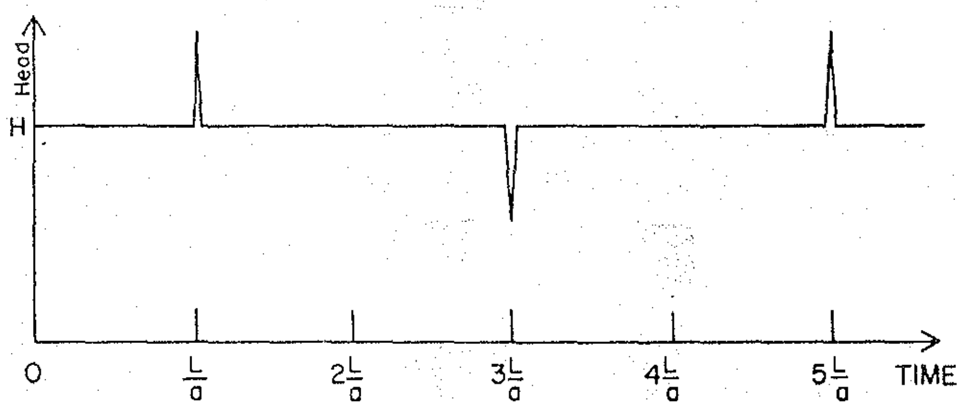


a) PRESSURE HEAD VS. TIME AT THE VALVE.

Fig 14



b) PRESSURE HEAD VS. TIME AT THE MIDPOINT.



c) PRESSURE HEAD VS. TIME AT THE RESERVOIR.

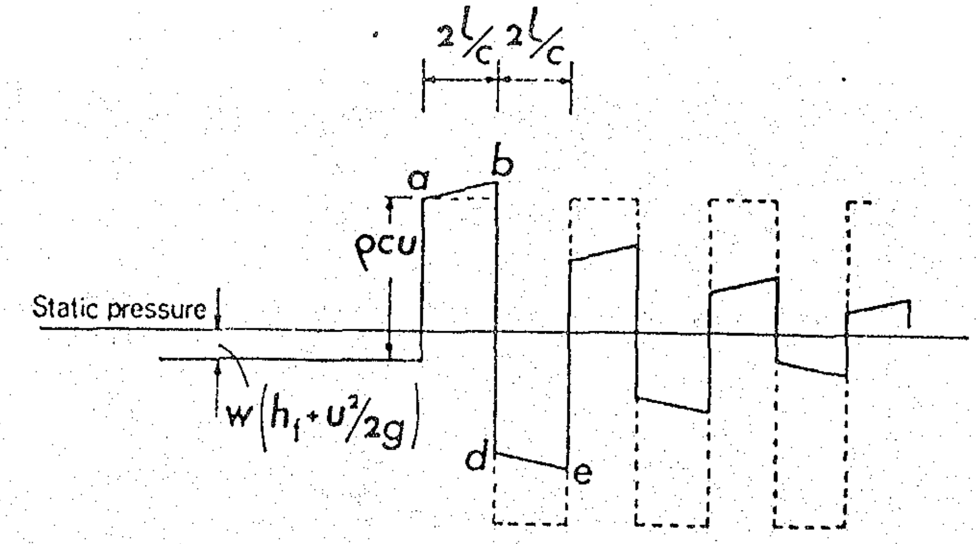


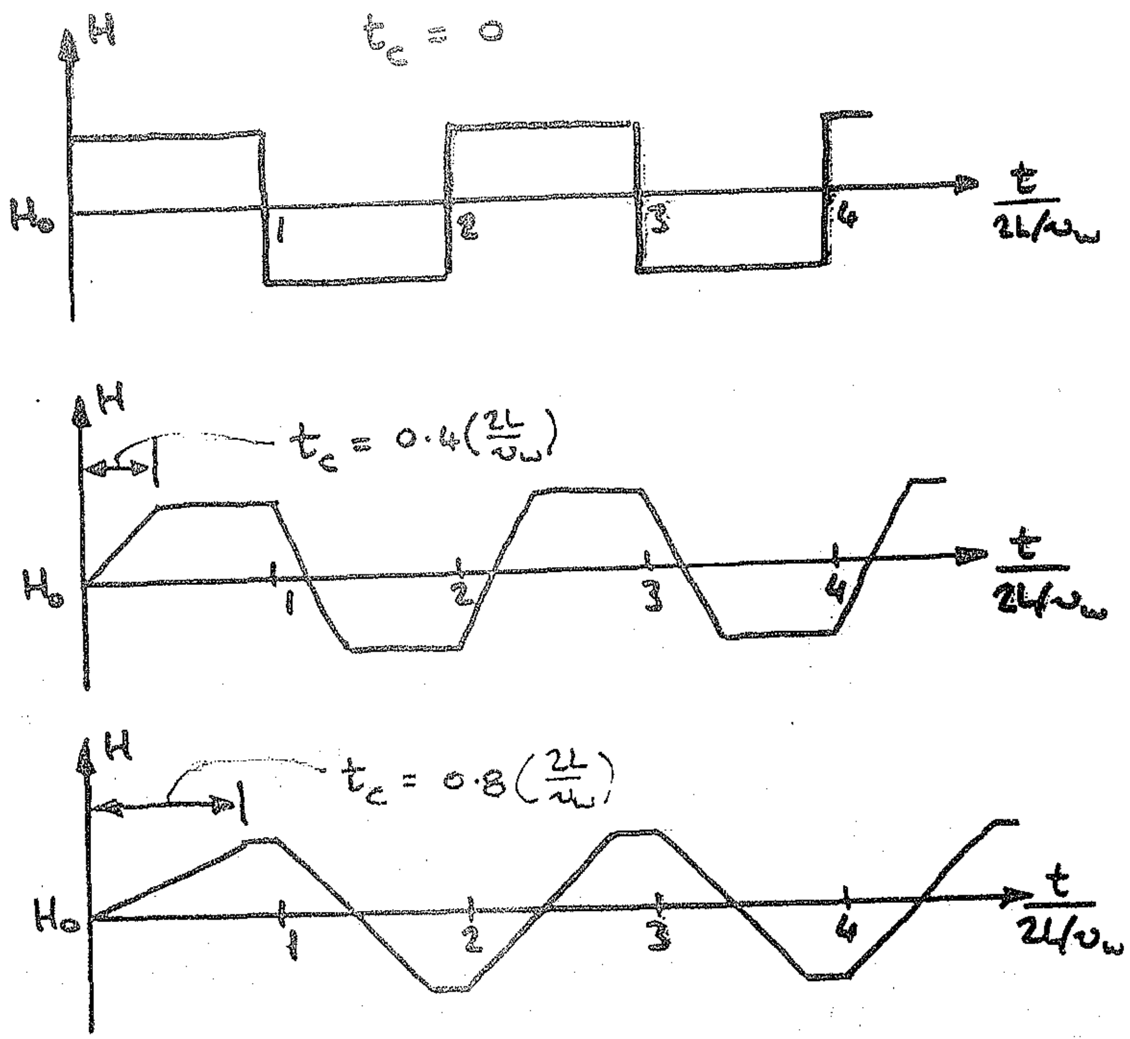
Fig 15 Effect of friction. Variation of pressure next to the valve with time after complete valve closure. Theoretical (no friction) line shown dotted.

The effects of friction losses are indicated in Figure 15, which shows the perhaps surprising fact that a greater rise of pressure may occur with friction than without it. When the velocity of the fluid is reduced, so is the head lost to friction, the head available at the downstream end of the pipe consequently rises somewhat as layer after layer of the fluid is slowed down. This secondary effect is transmitted back from each layer in turn with celerity a, and so the full effect is not felt at the valve until a time 2L/a after its closure. In the Figure this effect is indicated by the upward slope of the line ab. During the second time interval of 2L/a velocities and pressure amplitudes have reversed signs, and thus the line de slopes slightly downwards. However energy is also dissipated by viscous forces during the small movements of individual particles as the fluid is compressed and expanded. This dissipation of energy, known as damping, always tends to reduce the amplitude of the pressure waves.

Figure 16 shows the change in pressure distribution with time due to closure times less than 2L/a (instantaneous closure), while Figure 17 shows the situation for closure time between 2L/a and 5 x 2L/a (intermediate closure). In the latter case, the closure time is treated as a series of incremental closures each of duration 2L/a accumulated algebraically.

"INSTANTANEOUS" CLOSURE ($t_c \le \frac{2L}{a_w}$)

Fig 16



Intermediate Closure ($2L/v_w < t_c < 5\,(2L/v_w)$)

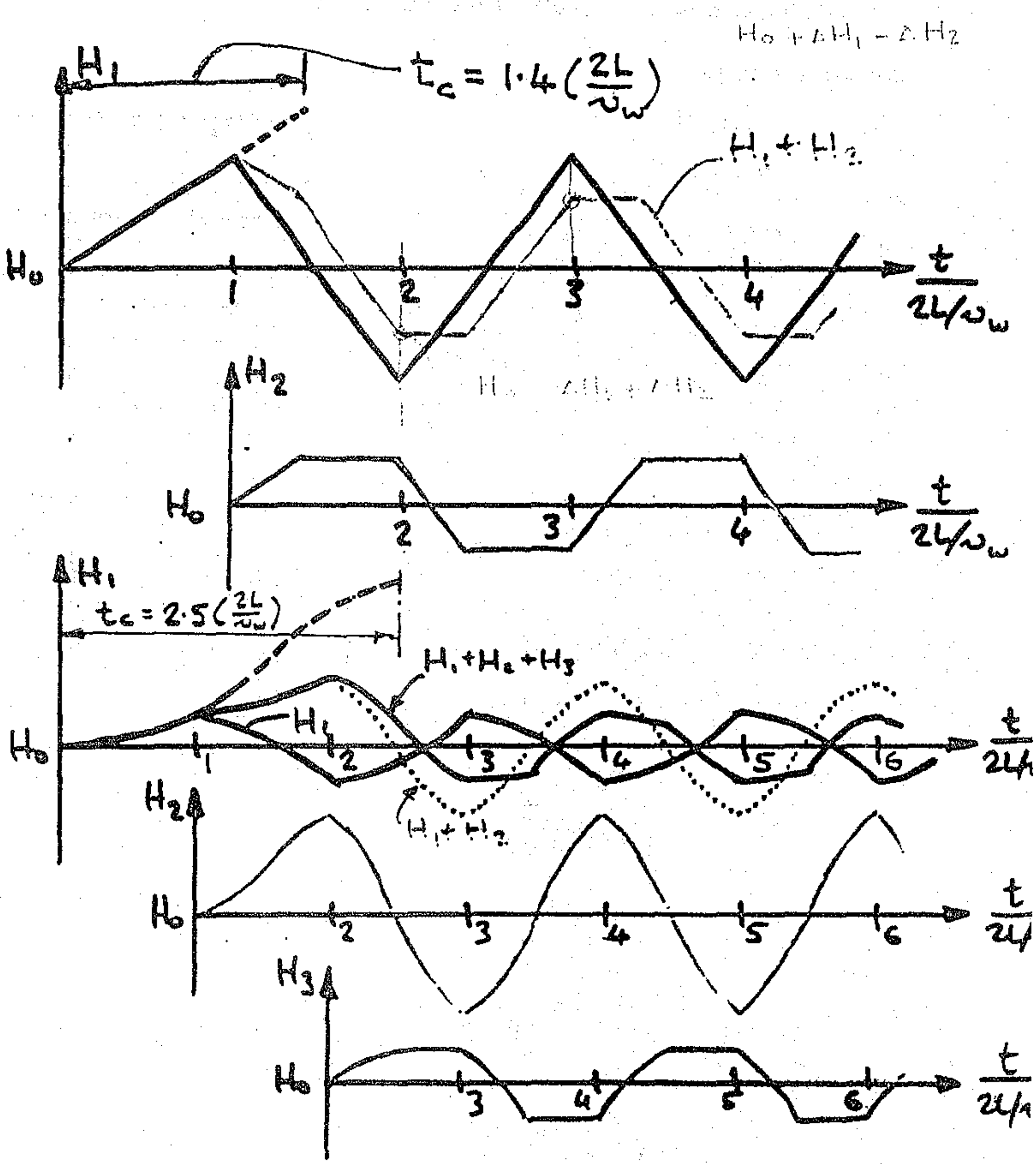


Fig 17

Wave Celerity c

Due to pressure increase ΔH, volume reduction is due to,

i) elastic compression

$$K = \frac{\Delta p}{\Delta Vol / Vol} = \frac{\rho g \Delta H}{\Delta Vol / A\Delta x}$$

therefore,

$$\Delta Vol_{ec} = (A\Delta x)\frac{\rho g \Delta H}{K}$$

ii) radial strain

$$E = \frac{stress}{strain} = \frac{\frac{pD}{2T}}{\frac{\Delta D}{D}} \quad hence \quad \Delta D = \frac{\rho g \Delta H D^2}{2ET}$$

therefore,

$$\Delta Vol_{rs} = \pi D \Delta x \frac{\Delta D}{2} = \frac{\rho g \Delta H}{ET}\frac{\pi D^3}{4}\Delta x$$

then the volume for incoming fluid is,

$$\Delta Vol = \rho\, g\, A\,\Delta x\,\Delta H\left[\frac{1}{K} + \frac{D}{ET}\right] = \rho\, g\, A\,\Delta x\,\Delta H\,\frac{1}{K_e}$$

where K_e is the equivalent Bulk modulus.

But $\Delta Vol = -\Delta V\ A\ \Delta t = \rho\, g\, \Delta H (A\,\Delta x)\dfrac{1}{K_e}$

Then

$$\frac{\Delta V}{\Delta x} = -\frac{\rho\, g}{K_e}\frac{\Delta H}{\Delta t} \quad \text{continuity equation}$$

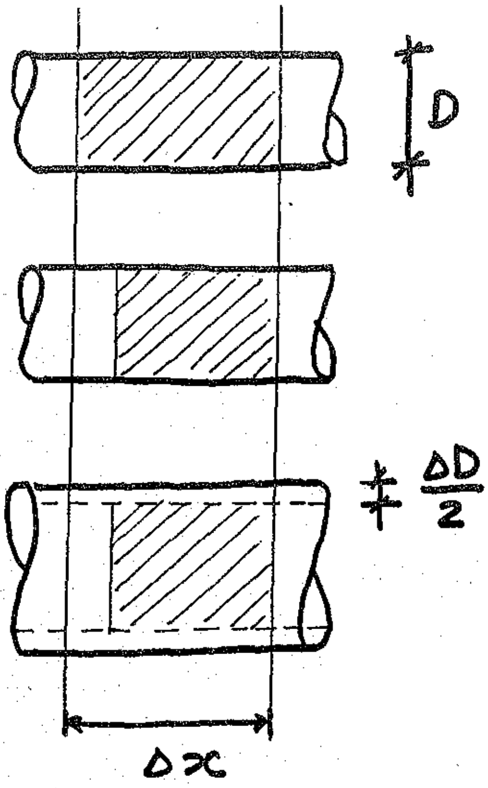


Fig 9

Recalling the dynamic equation developed before for unsteady flow

$$\frac{\Delta H}{\Delta x} = -\frac{1}{g}\frac{\Delta V}{\Delta t} = -\frac{1}{g}\frac{\Delta V}{\Delta x}\frac{\Delta x}{\Delta t} = \left(-\frac{1}{g}c\right)\left(-\frac{\rho g}{K_e\left(\frac{\Delta t}{\Delta H}\right)^{-1}}\right)$$

hence

$$c = \sqrt{\frac{K_e}{\rho}} \qquad \text{which is the equation for wave celerity}$$

Velocity of wave propagation is not a velocity with which particles of matter are moving, therefore we use the word 'celerity'. It represents the rate at which a 'message' can be 'telegraphed' through the fluid. If we ignore the elasticity of the pipe, then

$$c = \sqrt{\frac{K}{\rho}} \quad \text{which is the velocity of sound in an infinite expanse of liquid}$$

Example 7

A pipeline has a diameter of 300 mm and wall thickness 8 mm. If K = 2.05 GPa, and E = 200 GPa, find the vave celerity c.

$$\frac{1}{K_e} = \frac{1}{K} + \frac{D}{ET}$$

$$\frac{1}{K_e} = \frac{1}{2.05} + \frac{300}{200 \times 8}$$

K_e = 1.481 GPa

$$c = \sqrt{\frac{1.481 \times 10^9}{1000}} = 1217\ m/s$$

Example 8

Pipeline of length 1500 m has diameter of 300 mm and wall thickness of 8 mm steel, the valve closes flow of 0.15 m^3/s in 2 sec. Find ΔH.

From the previous example K_e = 1.481 and c = 1217 m/s,

$$t_r = \frac{2L}{c} = \frac{2 \times 1500}{1217} = 2.465 \text{ sec} \rangle\ t_c = 2 \text{sec}, \textit{ then closure is instantaneous}$$

$$V = \frac{0.15}{\frac{\pi \times (0.3)^2}{4}} = 2.122\ m/s$$

$$\Delta H = -\frac{1217}{9.81}(0 - 2.122) = 263\ m$$

Example 9

If in the previous example closure reduces α from 1.0 to 0.3 in 2 sec, find H_{max}, if H_o = 120 m.

$$V_f = 0.3 \times 2.122 \times \sqrt{\frac{120 + \Delta H}{120}} \qquad (1)$$

$$\Delta H = -\frac{1217}{9.81}(V_f - 2.122) \qquad (2)$$

Solving (1) and (2), we get V_f = 0.947 m/s, and ΔH = 145.7 m.

H_{max} = 120 + 145.7 = 265.7 m.

Intermediate Closure

When valve closure extends beyond 2L/c, expansion wave returns to valve before closure is completed and reduces the total inertia pressure. As mentioned above this closure is treated as a series of incremented closures each of duration 2L/c. Each incremental closure produces a wave of pressure. These are accumulated algebraically. For each time increment $\Delta t = 2L/c$,

$$\Delta H_i = -\frac{c}{g}\Delta V_i$$

$$V_i = \alpha_i V_o\sqrt{\frac{H_i}{H_o}}$$

$$\Delta V_i = V_i - V_{i-1}$$

$$H_i = H_o + \Delta H_i - \Delta H_{i-1} + \Delta H_{i-2} - \cdots$$

At $t = 1 \times 2L/c$ $\quad H_1 = H_o + \Delta H_1$

$t = 2 \times 2L/c$ $\quad H_2 = H_o + \Delta H_2 - \Delta H_1$

$= H_o + \Delta H_2 - H_1 + H_o$

$= 2\,H_o + \Delta H_2 - H_1$

$t = 3 \times 2L/c$ $\quad H_3 = H_o + \Delta H_3 - \Delta H_2 + \Delta H_1$

$= H_o + \Delta H_3 + 2\,H_o - H_2 - H_1 + H_1 - H_o$

$= 2\,H_o + \Delta H_3 - H_2$

In general $\quad \underline{H_i = 2H_o + \Delta H_i - H_{i-1}}$

Example 10

For L = 1500 m, V_o = 2.3 m/s, H_o = 120 m, and closure to α = 45% in time of 8 sec. Use E = 200 GPa, K = 2.08 GPa, D = 300 mm, and T = 7 mm, Find the maximum head at the valve.

1. Find K_e and c

$$\frac{1}{K_e} = \frac{1}{2.08} + \frac{300}{200 \times 7} = 0.6951 \quad \Rightarrow K_e = 1.439\ GPa$$

$$c = \sqrt{\frac{1.439 \times 10^9}{1000}} = 1200\ m/s$$

2. Calculate reflection time

$$\frac{2L}{c} = \frac{2 \times 1500}{1200} = 2.5\ \sec \le t_c$$

Subdivide closure

t_1 = 2.5 sec

$$\alpha_1 = 1.0 - \frac{(1.0 - .045)}{8} \times 2.5 = 0.8281$$

t_2	5 sec	α_2	0.656
t_3	7.5 sec	α_3	0.4844
t_4	10 sec	α_4	0.450

3. Solve for H_i= 1,2,3,4

$$V_1 = 0.8281 \times 2.3 \sqrt{\frac{H_1}{120}} = 0.1738 \sqrt{120 + \Delta H_1}$$

$$\Delta H_1 = -\left(\frac{1200}{9.81}\right) \Delta V_1 = 122.32\,(2.3 - V_1)$$

$$V_1^2 = 0.0302\,(120 + 281.34 - 122.32\,V_1)$$

$$V_1^2 + 3.698\,V_1 - 12.132 = 0 \Rightarrow V_1 = 2.095\ m/s$$

$$\Delta H_1 = 25.13\ m \Rightarrow H_1 = 145.13\ m$$

Second increment

$$V_2 = 0.6563 \times 2.3 \sqrt{\frac{H_2}{120}} = 0.1378 \sqrt{120 + \Delta H_2 - \Delta H_1}$$

$$\Delta H_2 = -\left(\frac{1200}{9.81}\right) \Delta V_2 = 122.32\,(2.0945 - V_2)$$

$$V_2^2 + 3.3226\,V_2 - 6.661 = 0 \Rightarrow V_2 = 1.6697\ m/s$$

$$\Delta H_2 = 122.32(2.0945 - 1.6697) = 51.96\ m$$

$$H_2 = 120 + 51.69 - 25.13 = 146.83\ m$$

Third increment

V_3 = 1.2332 m/s, ΔH_3 = 53.56 m, H_3 = 146.71 m

Fourth increment

V_4 = 1.028 m/s, ΔH_4 = 24.99 m, H_4 = 118.28 m

The attached spreadsheet demonstrates the solution to the problem in addition to calculating the effect of changing the closure time t_c on the maximum head H_{max}.

Similar to Example 5, the coefficients of the quadratic equation used to solve for V_i is given by the following expressions,

$$b = \frac{(\alpha_i\, v_o)^2}{H_o} \frac{c_w}{g}$$

$$c = -\left[2(\alpha\, v_{i-1})^2 + b\,\frac{c_w}{g}\, v_{i-1} - \frac{H_{i-1}}{H_0}\right]$$

Valve Opening

The same solution procedure can be used to simulate the valve opening operation. First, the valve movement has to be checked if instantaneous or not. The velocity and head are calculated as in the case of intermediate closure to account for the returning wave. The following example illustrates the solution steps.

Example 11

For L = 1500 m, V_o = 2.3 m/s, H_o = 120 m, and valve opens from $\alpha_o = 0$ to α_f = 50% in time of 2 sec. Using a wave speed of 1200 m/s, find the maximum and minimum head at the valve.

1. Calculate reflection time

$$\frac{2L}{c} = \frac{2\times 1500}{1200} = 2.5 \text{ sec} \geq t_c$$

Valve movement is instantaneous

2. Solve for H_i= 1,2,3,4

At time 2.5 sec

$$V_1 = 0.5\times 2.3 \ \frac{H_1}{120} = 0.105 \ \ 120+\Delta H_1$$

$$\Delta H_1 = -\left(\frac{1200}{9.81}\right)\Delta V_1 = 122.32\,(V_1 - 0)$$

$$V_1^2 = 1.3225 - 1.3481\,V_1$$

$$V_1^2 + 1.3481V_1 - 1.3225 = 0 \ \Rightarrow \ V_1 = 0.659\ m/s$$

$$H_1 = 120 - 122.32\ (0.659 - 0.0) = 39.4\ m$$

At time 5 sec

$$V_2 = 0.105\sqrt{H_2}$$

$$\Delta H_2 = -122.32\,(V_2 - 0.659)$$

$$V_2^2 = 0.01102\,(120\times 2 - 122.32\,(V_2 - 0.659) - 39.4)$$

$$V_2^2 + 1.3481V_2 - 3.099 = 0 \ \Rightarrow \ V_2 = 1.211\,m/s$$

$$\Delta H_2 = -122.32\,(1.211 - 0.659) = -67.52\ m$$

$$H_2 = 120\times 2 - 67.52 - 39.4 = 133.08\ m$$

At time 7.5 sec

V_3 = 1.133 m/s, ΔH_3 = 9.54 m, H_3 = 116.47 m

At time 10 sec

$V_4 = 1.154$ m/s, $\Delta H_4 = -2.61$ m, $H_4 = 120.91$ m

The attached spreadsheet demonstrates the solution to the problem.

Similar to Example 10, the coefficients of the quadratic equation used to solve for V_i is given by the following expressions,

$$b = \frac{(\alpha_i\, v_o)^2}{H_o}\frac{c_w}{g}$$

$$c = -\left[2(\alpha_i\, v_o)^2 + b\frac{c_w}{g}v_{i-1} - \frac{H_{i-1}}{H_o}\right]$$

Pressure at Intermediate Points

All examples so far consider pressure at the valve, i.e. at farthest point from reservoir. For intermediate points reflection time 2L/c is reduced and type of closure may vary.

For example for L = 1500 m and c_w = 1200 m/s, the reflection time is $2L/c_w$ = 2.5 sec. If t_c = 2 sec closure is instantaneous.

- For a point 300 m upstream from the valve, L' = 1200 m, and $2L/c_w$ = 2 sec, then closure is still instantaneous.
- For a point closer to reservoir, 600 upstream the valve, L" = 900 m, and $2L/c_w$ = 1.5 sec, then closure is intermediate.

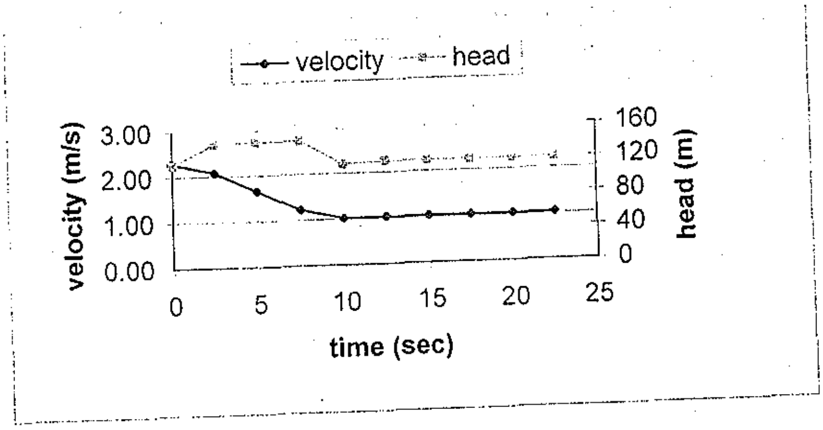
Example 12

For the following data,
L = 1500 m
c_w = 1200 m/s
H_o = 120 m
V_o = 2.3 m/s
α_o = 1.0
α_f = 0.3
t_c = 2 sec

Find the maximum head at points along the pipe every 300 m.

Elastic Water Column

	Complete Closure		Incomplete closure
Input Data			
Ho	120 m	intial pressure head at valve	
D	300 mm	pipe diameter	
T	7 mm	pipe wall thickness	
E	200 GPa	modulus of elasticity	
K	2.08 GPa	bulk modulus	
ρ	1000 kg/m3	density of liquid	
g	9.81 m/s2	acceleration of gravity	
L	1500 m	length of pipe	
Q	0.15 m3/s	flow	
tc	2 sec	closure time	2 sec
αo	1	initial valve area ratio	1
αf	0	final valve area ratio	0.3
Intermediate Calculations			
Ke	1.439 GPa	equivalent bulk modulus	
Vw	1199 m/s	wave speed	
tr	2.501 sec	reflection time	
Type of closure		instantaneous	
Vo	2.300 m/s	initial velocity	
Vf	0.000 m/s		1.042 m/s
ΔH	281.2 m	increase in pressure at valve	153.8 m



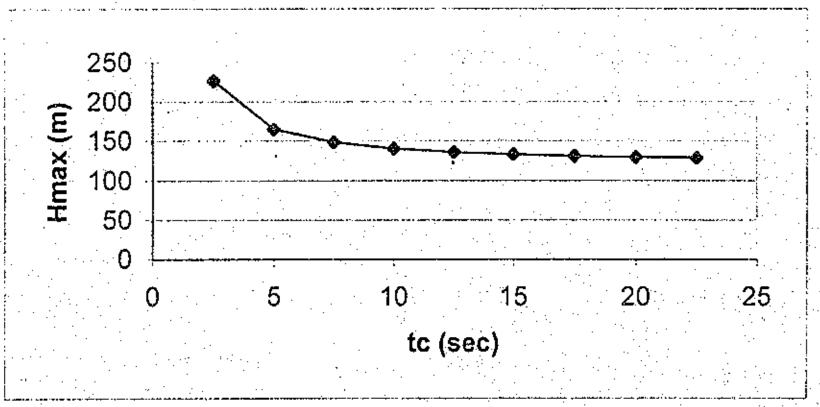
Intermediate Closure

Effect of Changing tc on Hmax

tc	146.85
2.5	227.1
5	165.8
7.5	148.8
10	141.0
12.5	136.5
15	133.6
17.5	131.6
20	130.1
22.5	128.9

8 sec
1
0.45

t (sec)	α	V	b	c	ΔH	H
0	1	2.30			0.00	120.00
2.5	0.828	2.09	3.70	-12.128	25.13	145.13
5.0	0.656	1.67	2.32	-6.660	51.98	146.85
7.5	0.484	1.23	1.26	-3.072	53.56	146.71
10.0	0.450	1.03	1.09	-2.177	24.94	118.22
12.5	0.450	1.04	1.09	-2.208	-1.23	120.55
15.0	0.450	1.03	1.09	-2.199	0.38	119.83
17.5	0.450	1.04	1.09	-2.202	-0.12	120.05
20.0	0.450	1.03	1.09	-2.201	0.04	119.98
22.5	0.450	1.04	1.09	-2.201	-0.01	120.01
					Hmax	146.85



The General Water Hammer Equations

$$\frac{\partial V}{\partial x} = -\frac{\rho g}{K_e}\frac{\partial H}{\partial t} \qquad \textit{continuity equation}$$

$$\frac{\partial H}{\partial x} = -\frac{1}{g}\frac{\partial V}{\partial t} \qquad \textit{dynamic equation}$$

If we differentiate the first equation wrt to t and and second wrt to x we obtain

$$\frac{\partial^2 V}{\partial x \partial t} = -\frac{\rho g}{K_e}\frac{\partial^2 H}{\partial t^2}$$

$$\frac{\partial^2 H}{\partial x^2} = -\frac{1}{g}\frac{\partial^2 V}{\partial x \partial t}$$

Or

$$\frac{\partial^2 H}{\partial t^2} = \frac{K_e}{\rho}\frac{\partial^2 H}{\partial x^2} \quad \textit{similarly} \quad \frac{\partial^2 V}{\partial t^2} = \frac{K_e}{\rho}\frac{\partial^2 V}{\partial x^2}$$

These are plane wave equations in H and V with respect to x and t, where $K_e/\rho = c^2$. The general solution to the equations is,

$$H - H_o = F(t - \frac{x}{c}) + f(t + \frac{x}{c})$$

$$V - V_o = -\frac{g}{c}\left[F(t - \frac{x}{c}) - f(t + \frac{x}{c})\right]$$

The physical significance of F(t-x/c) is a wave of constant shape moving in +ve direction with velocity c. After time Δt the wave moves a distance, $\Delta x = c\,\Delta t$, therefore

$$F(t + \Delta t - \frac{x + \Delta x}{c}) = F(t - \frac{x}{c} + \Delta t - \frac{\Delta x}{c}) = F(t - \frac{\Delta x}{c})$$

if and only if $\Delta x/c = \Delta t$.

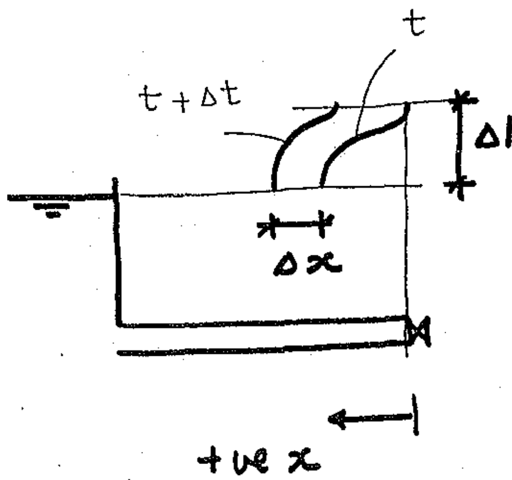


Fig 10

Similarly, f(t+x/c) describes a wave of arbitrary by constant shape moving in -ve direction with velocity c.

The following uses the mathematical solutions to explain the reflection of the pressure wave at the upstream and downstream boundaries.

i) case 1 open boundary
$H = H_o$ = constant at $x = L$

$$F(t-\frac{x}{c})+f(t+\frac{x}{c})=H-H_o=0,$$

$$hence\ f(t+\frac{x}{c})=-F(t-\frac{x}{c})$$

i.e. wave F() is reflected with opposite sign, e.g. compression wave is reflected as expansive wave. Also, at time t=L/c

$$V-V_o=-\frac{g}{c}\left[F(t-\frac{L}{c})-f(t+\frac{L}{c})\right]$$

$$=-\frac{2g}{c}F(t-\frac{L}{c})$$

i.e. velocity change is doubled in magnitude, thus when pipeline is under ΔH over entire length V=0 but when expansion wave travels down velocity reverse and flow returns to reservoir.

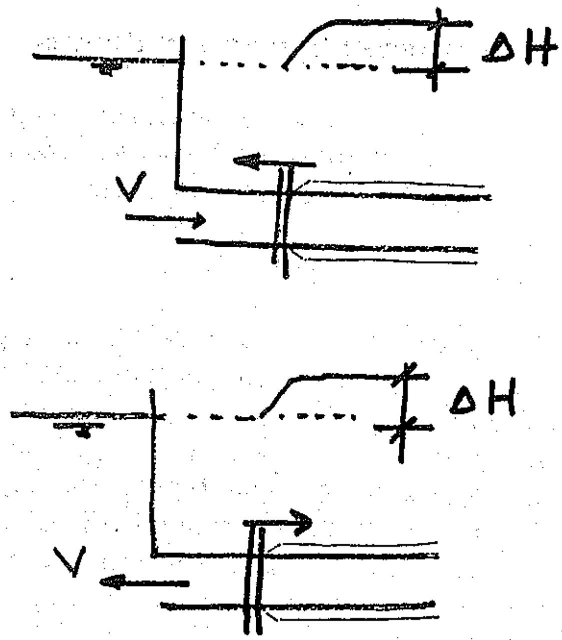


Fig 11

ii) case 2 closed boundary
If downstream valve is closed ΔV=0, thus

$$F(t-\frac{x}{c})-f(t+\frac{x}{c})=0,$$

$$hence\ H-H_o=2\,F(t-\frac{x}{c})$$

i.e. wave is reflected with no change in sign, e.g. wave of expansion continues as a further wave of expansion.

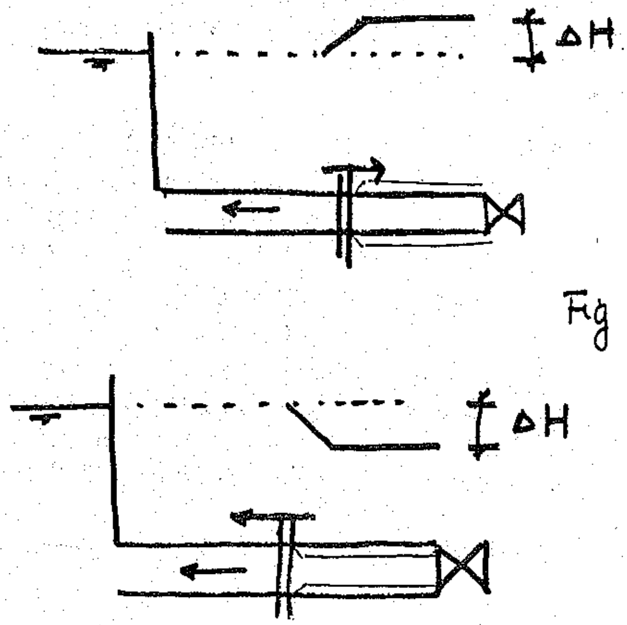


Fig 12

Water Hammer Effects and Control

This section includes a discussion on the effects of pump stop, power failure, and water column separation, and the control devices like valves, air in lines, surge tanks, and other techniques.

Effects

If the pump motor is shut off the pump goes down in speed, and when the pump goes down in speed, a pressure decrease is created at the pump, and propagates as the front of a *sub pressure wave* ΔH in the pipeline. After a time period $t = L/c$ the wave reaches the outlet of the pipeline and is reflected back towards the pump, but a *pressure wave* (as shown in Figure 18). By comparing the minimum pressure line and the pipe profile, we can find the out where the lowest pressures occur in the pipeline. These lowest pressure zones cause pipe buckling or collapse and fatigue for an on-off regulated system.

If the system has a swing type check valve, it closes at the moment of flow reversal, where the flow is still towards the closed check valve. The moving water compress against the check valve causing pressure to rise, called *upsurge pressure*. The latter pressure could rise up to twice the value of the static head.

When negative pressures along the pipeline during downsurge are severe, it cause the water to vaporize along some portions of the pipeline, causing water column to separate and a vapor pocket to form (water column separation). It is at the high points that water column separation takes place during abrupt pump stoppage (as shown in Figure 19). The static head above the point of water column separation is the driving force behind the movement of the water column, causing the water column to accelerate to a velocity that is suddenly extinguished at the moment the vapor pocket collapses (water column collapse). It is not uncommon for the pressure at the site of column collapse to rise from –32 ft to 500 ft in about one second.

Control Devices and Techniques

1 Because the change in pressure is directly proportional to the change in velocity, avoiding sudden velocity changes generally prevents serious water hammer pressures. Most control devices and techniques are designed to function in a particular application to satisfy this generalization.

2 1 **Valves**

Controlled Valve Movement

It can be illustrated that the valve closing schedule could affect the maximum pressures developed, e.g. the last 2-5 percent or so of valve closure is the most critical for gate valves. By introducing controls onto the valve closure

Figure 18a.
Propagation of sub-pressure (negative) wave.

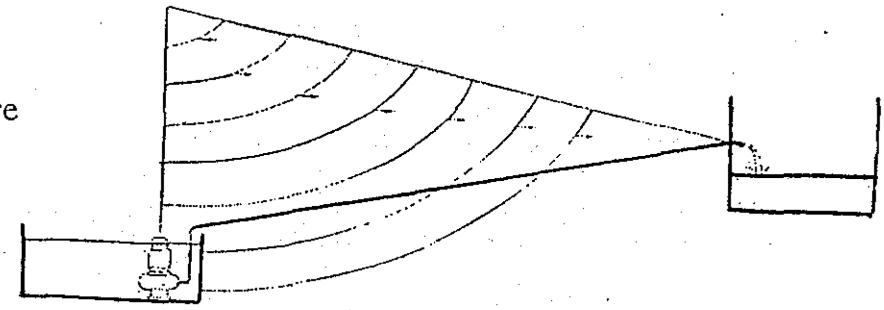
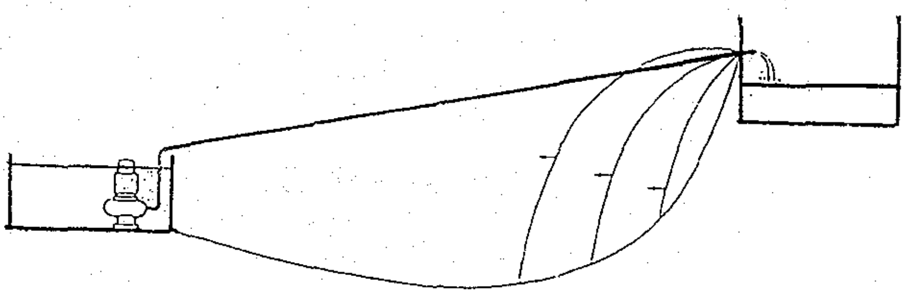


Figure 18b.
Propagation of reflected Pressure wave.



Minimum Pressure Line

T is time for water column to lose momentum and stop forward movement.

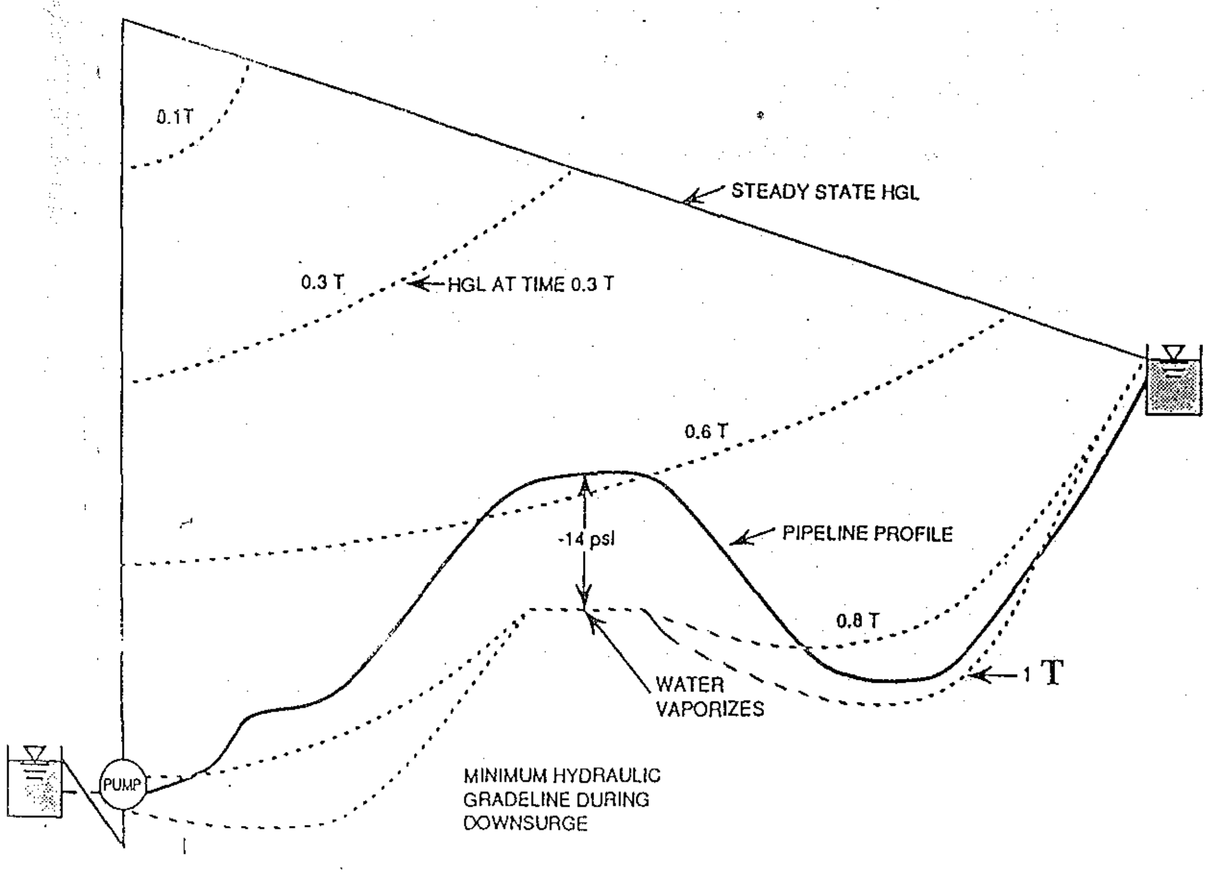


Figure 19 Down surge hydraulic grade line and column separation.

mechanism, particularly to slow down the final stages of closure, considerable reductions in surge pressure can be achieved.

a Slow Operating Valves

Slow operating valves (closing and opening) are installed at the discharge of pumps, and hence are often referred to as pump control valves. For water applications, butterfly valves are commonly used. For controlling water pressures during normal pump stoppages, slow operating valves are very effective.

b Surge Relief Valves

These valves designed to open when a certain prescribed pressure is exceeded, range from relatively inexpensive spring-loaded devices, to rather expensive and complicated systems. They are generally located adjacent to the device expected to cause the high pressure. The purpose of such valves is to provide an escape for the flowing liquid and prevent sudden decreases in velocity and the resulting high pressures. Some pressure relief valves are self regulating during closure (damped closure to allow for much longer closure time). During closing of the relief valve, the valve will halt and even reopen to limit pressure to the pre-set value in the event that the upsurge pressure reoccur. Figure 20 shows schematic diagrams of safety, relief and pressure regulating valves.

c By-pass Valving Systems

Figure 21 shows alternatives for protection of pumping stations: a) shows pump bypass to supply compensating flow if pump fails and low pressure regions forms on pump discharge, b) a check valve is installed to prevent back flow through the pump with a control valve in smaller bypass line, c) pump is isolated from the pipeline transient by a pump discharge-control valve, d) a system which allows to continue at a lower flow rate and minimizes the surge implications of local pump failure.

Air in Lines

Filling Empty Lines

Liquid is introduced slowly into the system at velocities of 1.0 fps or less, where release and air-vacuum valves must be provided so that the air can be forced from the system slowly. Another technique is to locate inexpensive spring-loaded pressure relief valves near the air valves to prevent the sudden velocity changes.

Removing Air from Lines

The best approach is to use the technique suggested for filling empty lines and not resume normal operation until all vacuum valves and air release valves have closed.

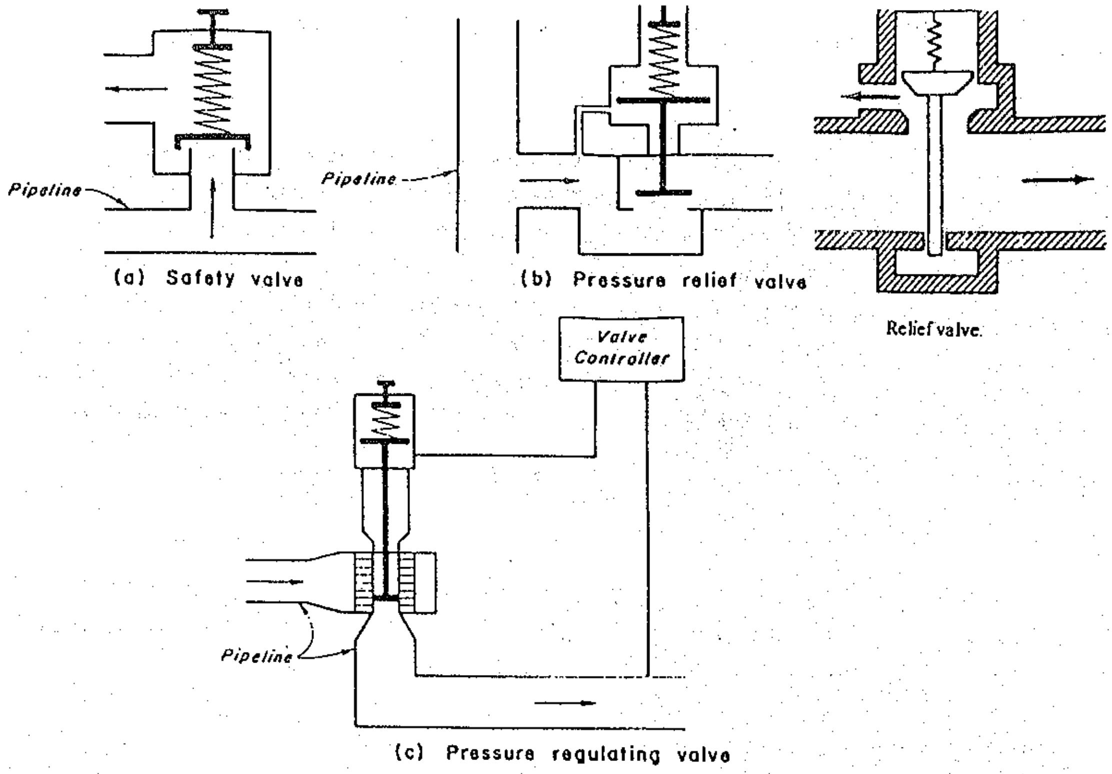


Figure 2 0 Schematic diagrams of safety, relief, and pressure regulating valves.

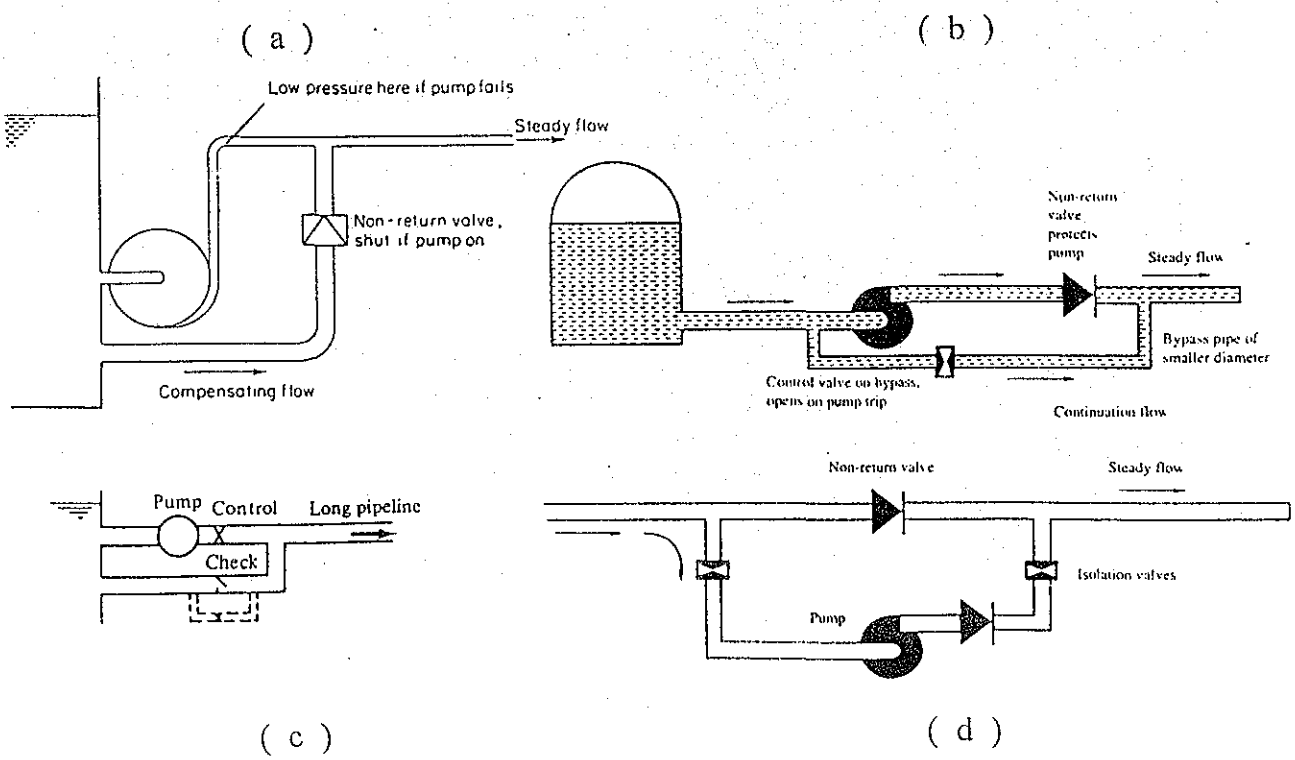


Figure 21 Pumping stations with by-pass systems

Surge Tanks

Can be employed to prevent both objectionably high and low pressures. They act as temporary storage devices for excess liquid which has been diverted from the main flow system to prevent high pressures, or they act as supplies of liquid to a pipe which needs more fluid to prevent excessive deceleration. Figure 22 illustrates some common surge tank (shaft) designs. For a one-way surge tank, if the hydraulic grade line drops below the surface of the liquid in the tank,a check valve opens and liquid flows into the system. The resulting hydraulic grade line configuration for a typical pumped pipeline with one-way surge tanks is shown in Figure 23.

Vented Surge Tanks

A variation on the one-way surge tank, which is sealed and equipped with a vacuum valve and air-release valve. This added capability can be most useful if reverse flow problems are anticipated.

Air Chambers

A relatively small pressurized vessel containing both water and air, as shown in Figure 24, connected to the main pipeline at the discharge side of pumps. In addition to suppressing surges, air chambers prevent negative pressures and column separation in the pipeline system downstream of the chamber. A typical situation is shown in Figure 25, which illustrates how gently the air chamber can bring the system to rest. The latter figure also shows that an air chamber is not always adequate to completely prevent column separation. In the case where low pressure can occur at local summits, a one-way surge tank is provided at the summit to 'drape' the HGL above the pipe as shown in Figure 26.

Wave Speed Reduction Methods

Wave speeds depend on (1) elastic wall properties an geometry, (2) liquid compressibility, and (3) free gas content in the pipeline. The response frequencies of complete systems may be altered by the wave speed change in component parts of the system, thereby eliminating resonance as a result of particular forced oscillation.

Bleeding in air

Entraining air into the liquid flow in a system is widely used to eliminate unwanted transients. The air is beneficial in reducing the severity of cavitation and also in reducing transient oscillations related to wavespeeds in the draft tubes.

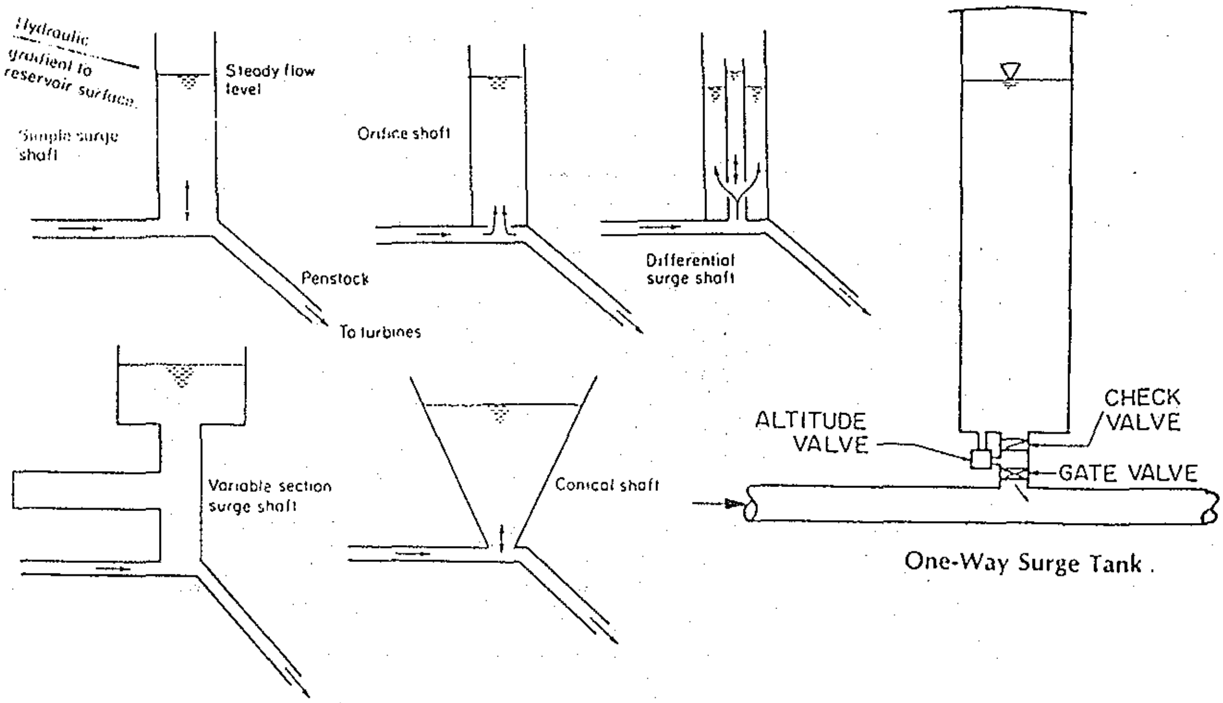


Figure 2.2 Schematics of some common surge tanks designs.

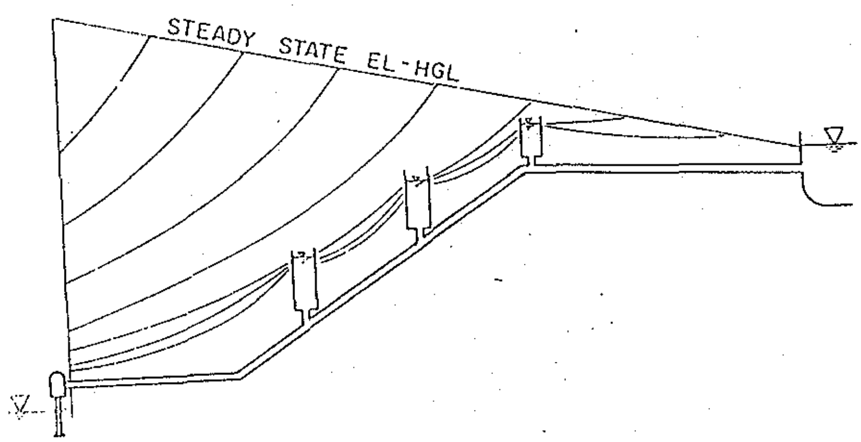


Figure 2.3 One-way surge tanks in a pumped pipeline.

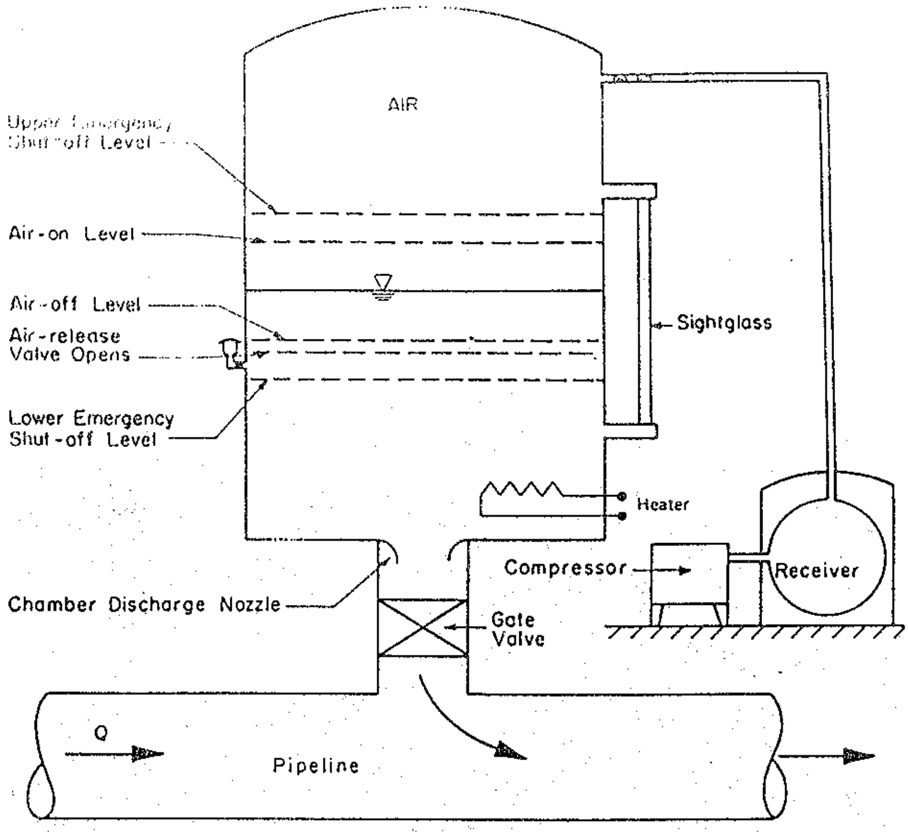


Figure 2.4 Schematic diagram of an air chamber and its appurtenances.

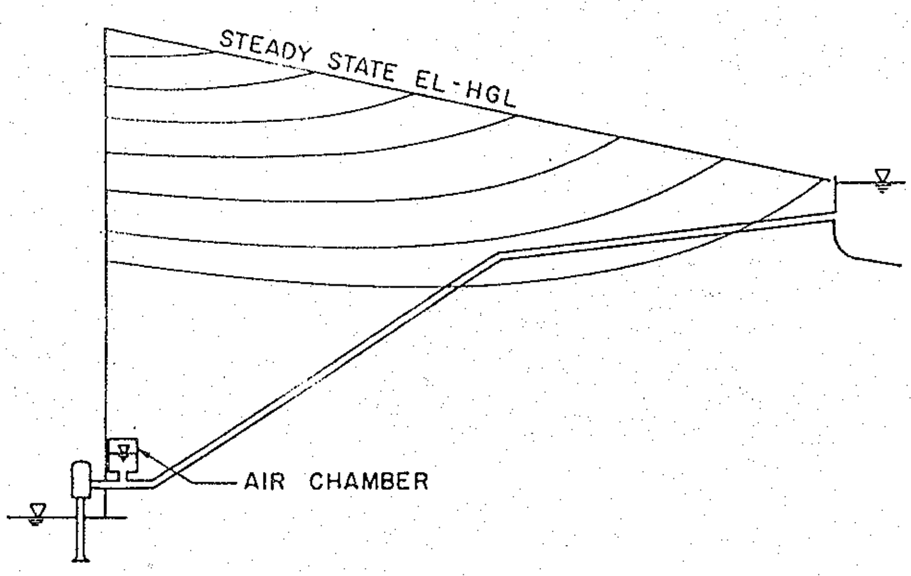


Figure 2.5 Propagation of negative wave after a pump power failure with an air chamber at the pump.

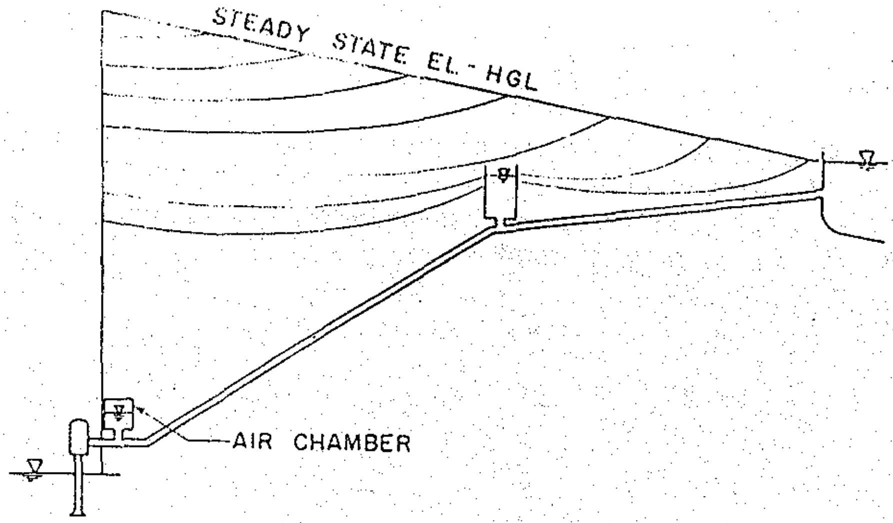


Figure 26 Propagation of negative wave after a pump power failure with an air chamber and a one-way surge tank..

Noncircular Conduits

A noncircular cylinder, when subjected to an internal pressure, is deformed generally into a shape more nearly approaching a circular cylinder, this increases the cross-sectional area per unit pressure change more than it would if the it were circular. The wave speed can be greatly reduced in this manner.

Flexible Hose

In many systems, flexible hoses (and many of the plastic hoses) are frequently employed as a means of reducing the severity of transients.

System Geometrical Design Changes

The transient performance of a piping system may be improved, in general, by increasing piping diameter. This design change may be particularly effective in suction lines, since it greatly decreases the possibility of cavitation. A dead-end branch (stub) originating at a chosen point along the system and of such a length that its response nullifies oncoming waves is very effective geometric change. A section of rubber or other flexible hose in a metal piping system generally increases natural periods of the system, and the magnitudes of head fluctuations are reduced accordingly. Also, stiffening and bracing the system at loop points of the pressure fluctuations reduces the harmful effect of resonance.

In Summary, by proper design of components such as valve controls, pipeline sizes, pump flywheels, etc. and by informed operational procedures, transient effects in fluid systems generally maybe controlled within satisfactory limits. The provision of an air chamber or surge tank is an excellent design option, followed in a priority ranking by other pressure- and flow-limiting devices as relief valves, check valves, air valves, bypasses with valving, etc.

References

1. Analysis and Control of Unsteady Flow in Pipelines, Gary Z. Watters, Butterworths, second edition, 1984.
2. Alan A. Smith, CE-304 Hydraulics course notes, Civil Engineering and Engineering Mechanics Department, McMaster University, Hamilton, Ontario, Canada, 1986.